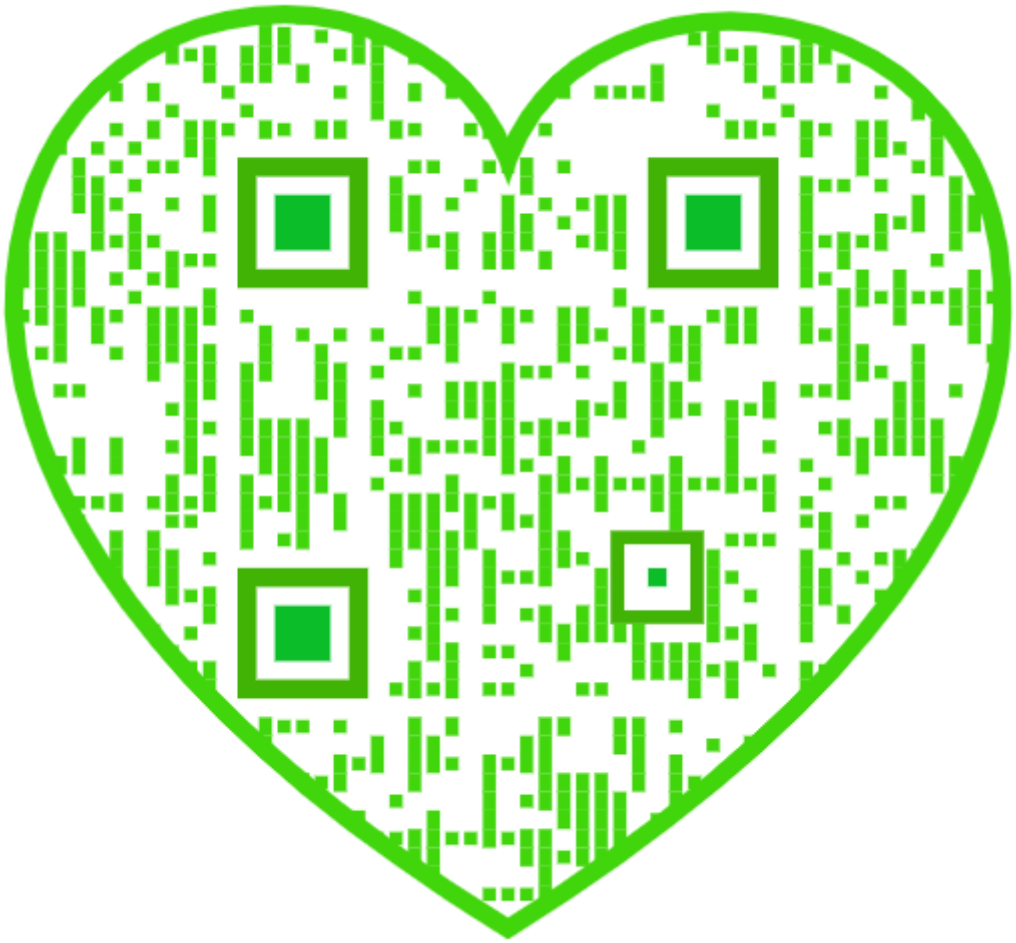

AMERICAN

ELEMENTARY ARITHMETIC

BY

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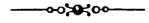

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E-P 1

PREFACE

THIS book is intended to cover the first five years' work in arithmetic; it is the first of a two-book series, of which the second is the "American Comprehensive Arithmetic." It is divided into two parts: Part First, for primary classes; Part Second, for the three succeeding grades.

The work is based upon the psychology of number. The need or demand which gives rise to each cumulative step in mental activity has been considered, and exercises have been prepared to develop this activity in harmony with the laws of the mind. Thus, after the pupil has learned how to find the value of $6 + 6 + 6 + 6$ by addition, his attention is called to the need of a shorter process, and he is introduced to multiplication by a method founded upon this demand. Psychology requires also that the relations of "what is given" and "what is required" shall be clearly ascertained as a basis for deciding upon the plan of procedure. These relations are not confined in this book to *ratio* alone; *i.e.*, to *how many times* one quantity contains another, but are extended more broadly to *how much* one quantity exceeds another.

It is now recognized that suitable apparatus is as necessary to the teacher of primary arithmetic as to the teacher of the physical sciences. The apparatus called for in this book consists of paper, pasteboard, etc., and of toy money, blocks, and splints, which may be prepared by the teacher, or obtained of any dealer in kindergarten materials.

Attention is called to the educative value of the illustrations. The attempt has been made to present every subject twice: first,

in the universal language of pictures; and second, in the particular form of printed words. Thus, on p. 29, the boys in the picture are seen to separate a whole of two objects into parts, and to unite the parts again into the whole. In printed words, the pupils are again instructed to perform the same operations.

In the first chapter, mathematical conceptions are developed in the order in which they arise in the consciousness of the child. At first, the emphasis is on mathematical judgments, relations, etc., without specific regard to number, or numbers as such. Thus, the first lesson develops the idea of "one or more than one" by questions based on the picture, such as, "Is there one girl or more than one?" Next comes a lesson on "how many by objects" with such questions as, "Hold up as many fingers as there are dolls on the floor." The next step is, "how many by words"; and the next, "how many by symbols." Then come in turn the ideas of "how much," "how much larger," "how many times," "how many times as large," "what part," and "what order." All this is introductory work designed to develop inductively, to present step by step a mathematical vocabulary, and to form a habit of clear mathematical thinking.

In succeeding chapters, the laboratory plan has been carefully followed. This plan has been carried out through the latest methods which have successfully met the test of actual trial in the schoolroom. In teaching the fundamental operations, the Grube method has not been pursued, but each operation has been so presented by itself that the teacher may follow the Grube method or any of its modifications by a judicious selection of the sequence of pages. The laws of apperception have been followed throughout. In every case, the pupil has been asked to consider what he already knows before he has been introduced to the unknown. The pupil has been taught to strive constantly for a view of the subject as a whole and to seek after general truths.

M. A. BAILEY.

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PART FIRST

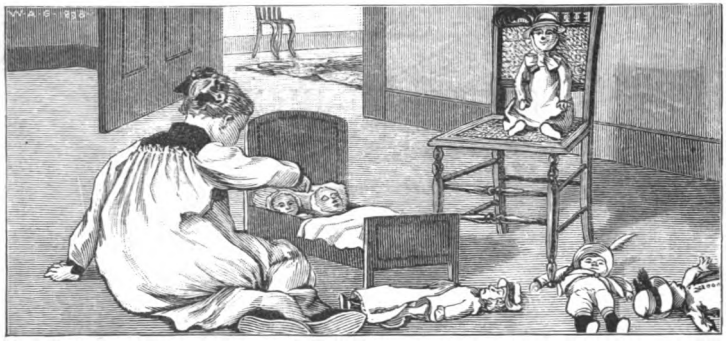
SIMPLE JUDGMENTS

One or More than One

Is there one girl or more than one?
Is there one doll or more than one?
Is there one boy or more than one?
Are there more boys than girls?
Are there more boys than dolls?
Are there more dolls than girls?
Are there more or fewer boys than girls?
Are there more or fewer girls than dolls?
Are there many or few bricks in the wall?
Are there many or few marbles in the ring?

NOTE. — The pupil is expected to answer such questions as these before he can read. With book in hand he may look at the picture, or he may be asked about other objects. These questions are meant to be suggestive of others.

How Many — By Objects



Hold up as many fingers:

As there are dolls on the chair.
As there are dolls in the bed.
As there are dolls on the floor.
As there are dolls with hats.
As there are dolls lying down.

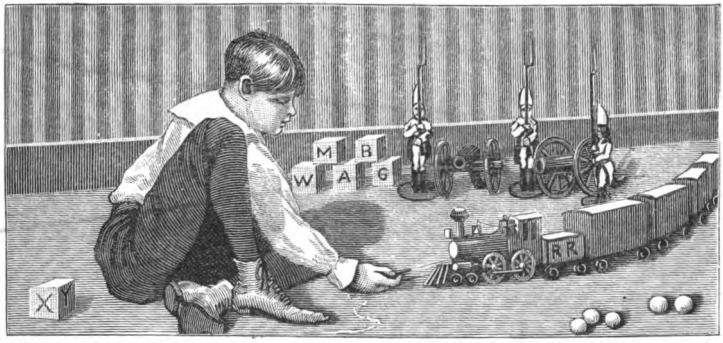
Place as many splints:

As there are dolls in the bed.
As there are dolls lying down.
As there are dolls on the floor.
As there are dolls sitting up.
As there are dolls with hats.

Make as many marks:

As there are dolls without hats.
As there are dolls with hats.
As there are dolls on the floor.
As there are dolls in the chair.
As there are dolls in all.

How Many — By Words



One boy, **1**
Two cannons, **2**
Three soldiers, **3**
Four cars, **4**
Five marbles, **5**
Six blocks, **6**

How many wheels has a wheelbarrow?

How many wheels has a bicycle?

How many eyes has a boy? How many eyes has a horse?

Name some other animal that has two eyes.

How many wheels has a carriage?

Name some animal that has four legs.

Can you name an animal that has three legs?

Can you name an animal that has three eyes?

You may bring me three books.

You may bring me three other objects of any kind.

You may make three marks on the board.

How many fingers have you on one hand, counting the thumb?

Let me hear you count the fingers of your hand.

Ans. Touch the thumb and say *one;* the index finger, and say *two*, and so on.

How Many — By Symbols

Make the figure for the number of boys planting melons.

Make the figure for the number of girls setting out tomato plants.

Make the figure for the number of men plowing.

Make the figure for the number of legs the horse has.

Make the figure for the number of hills of melons the boys have planted.

Make the figure for the number of tomato plants the girls have set out.

Let us play garden, and plant corn and beans.

Plant 3 hills of corn with 2 grains in each hill. Put them in a row.

Plant 3 hills of corn with 3 grains in each hill.

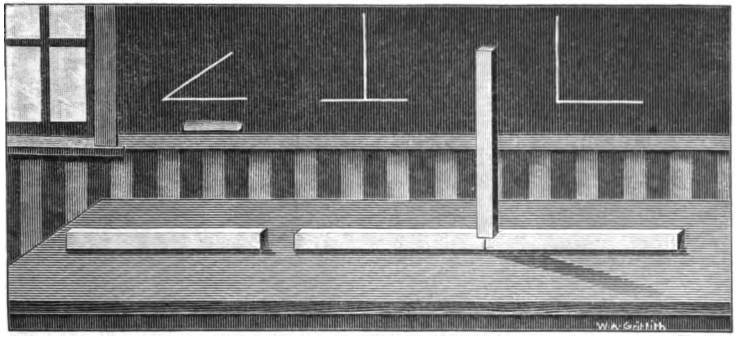
Plant 5 hills with 2 grains in each hill; plant 4 more rows like the one you have just planted.

Plant 4 hills with 3 beans in each hill.

Plant 5 hills with 4 beans in each hill.

Plant 3 hills with 2 beans in each hill.

Comparisons — Lines



Take splints and cut them of the same length as those in the picture; each is 1 inch long.

Draw a straight line 1 inch long; draw another of equal length.

Lay 2 one-inch splints in a line end to end.

Draw a line 2 inches long.

Lay 3 one-inch splints in a line. Draw a line 3 inches long.

Cut a string 4 inches long. Cut a string 5 inches long.

Put 2 splints together so as to make an opening. They form an angle.

Make another angle equal to the first.

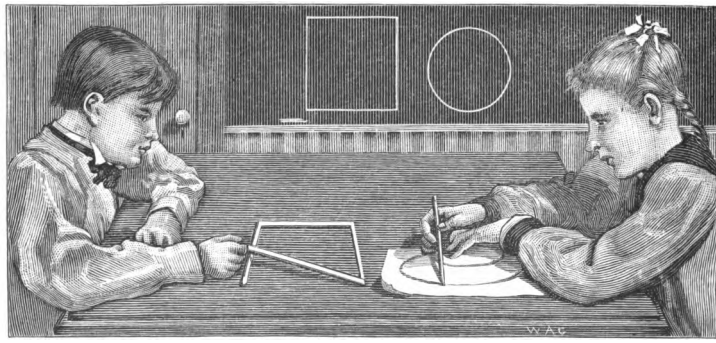
Draw 2 lines which shall form an angle.

Lay 2 splints in a line; put an end of another splint where the first two meet; make the angles equal. Each angle is a right angle.

Form a right angle with two splints.

Draw a straight line; from any point not in this line draw another which shall make right angles with it.

Comparisons — Squares and Circles



Take 4 equal splints and make a pen; make the angles all right angles. You have a square.

Draw a square; be sure to make all the lines equal and all the angles right angles.

Cut a square out of paper.

Make a square with 4 one-inch splints. What is the distance around it?

Lay a penny on a piece of paper and draw a line around it. You will have a circle.

Name something that resembles a circle in shape.

John has a garden in the shape of a square. Draw it.

Is a pane of glass in the form of a square? Is it almost a square?

Name some other objects that are almost in the shape of squares.

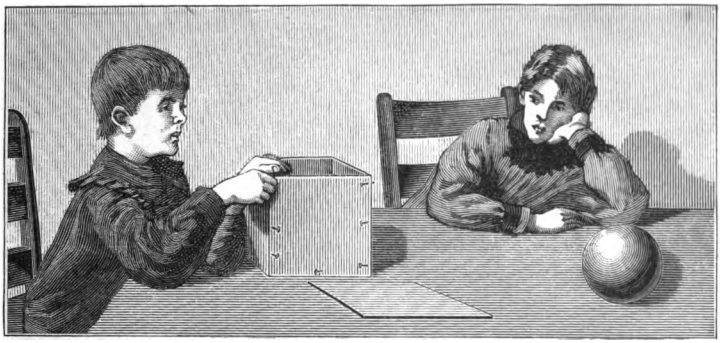
What are the parts called that bound a square?

Ans. Straight lines.

What is the part called that bounds a circle?

Ans. A curved line.

Comparisons — Cubes and Spheres



From pasteboard cut out 6 equal squares.

Make a box with them; fasten the pieces together with pins. You have a cube.

Is this room in the form of a cube? Is it almost a cube?

Is the chalk box in the form of a cube? Why not?

Name some object that is shaped almost like a cube.

Here is a baseball. It is called a sphere.

Is a marble a sphere. *Ans.* Yes.

Is a doll's head a sphere? *Ans.* It is almost a sphere.

Name some part of your body that is almost a sphere.

What is the boundary of a sphere? *Ans.* A curved surface.

What are the faces called that bound a cube? *Ans.* Squares.

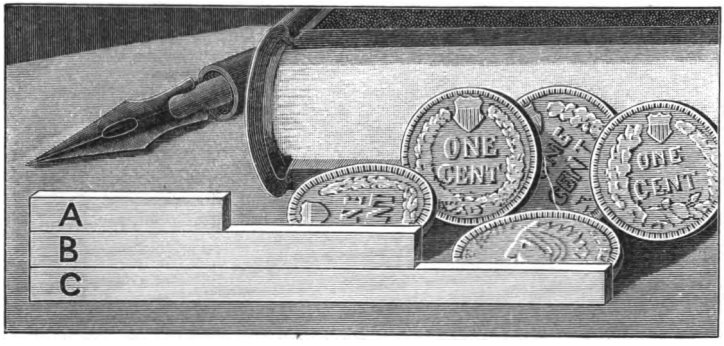
How many squares bound a cube? How do they compare in size?

What are the parts called that bound the squares on the cube? *Ans.* Edges, or lines.

How do the edges of a cube compare in length?

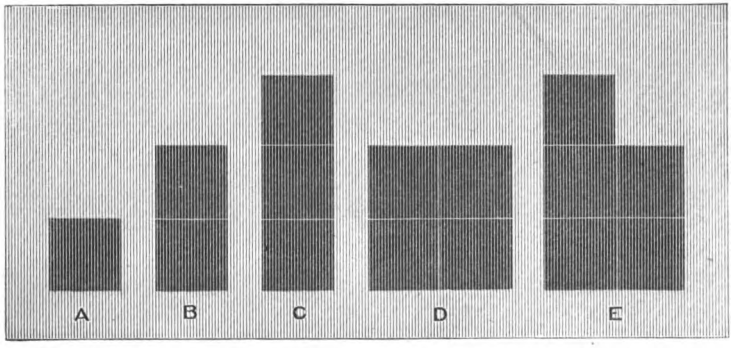
How do the edges of a chalk box compare in length?

How Much



How many inches long is A? *Ans.* 1.
How many inches long is B? *Ans.* 2.
How many inches long is C? *Ans.* 3.
How much is the length of A? *Ans.* 1 inch.
How much is the length of B? *Ans.* 2 inches.
How much is the length of C? *Ans.* 3 inches.
How many pennies are there in the picture?
How much money is there in the picture?
How many pennies did you pay for your pencil?
How much money did you pay for your pencil?
How much is one banana worth?
For how much can you purchase an orange?
How many glasses of water can you drink?
How much water can you drink?
How many cents make a nickel?
How much money is a nickel?
How many sticks of candy can you buy for a nickel?
How much candy can you buy for a nickel?
How many fingers have you on one hand?

How Much Larger



How many squares has C? How many squares has D? Which is the larger, D or C?

By how many squares is D larger than C? By how many is 4 more than 3?

How much larger is D than C? *Ans.* 1 square.

What is the relation of D to C?

Ans. D is one square larger than C.

How many squares has E? How many squares has B? Which is the larger, E or B?

By how many squares is E larger than B?

How much larger than A is C? By how many is 3 more than 1?

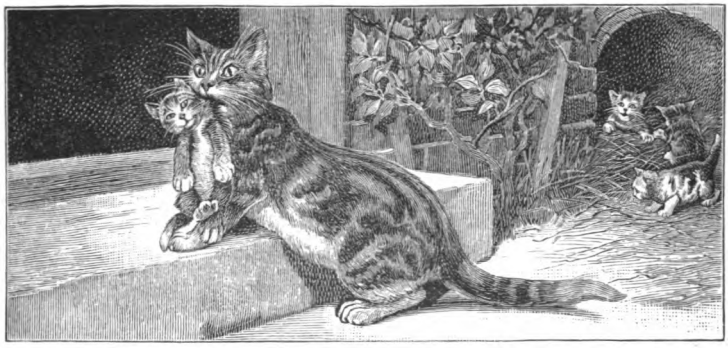
What is the relation of D to E?

Ans. D is 1 square smaller than E.

How much larger than C is E? By how many is 5 more than 3?

What is the relation of E to A? By how many squares is E larger than A?

How Many Times



How many times must the cat go into the house to carry in all her kittens?

How many times must a squirrel climb a tree to carry 5 nuts to his nest in a hollow limb, if he can carry only one nut at a time?

How many times a day do you sit at the table and eat?

How many times a day do you have recess?

How many times a day do you go to bed?

How many times a day do you drink water?

Can you jump on one leg five times?

Can you bounce a ball and catch it 4 times without missing?

Can you jump a rope 3 times without missing?

Raise your right hand three times.

Raise your left hand twice.

Step backwards twice and forwards three times.

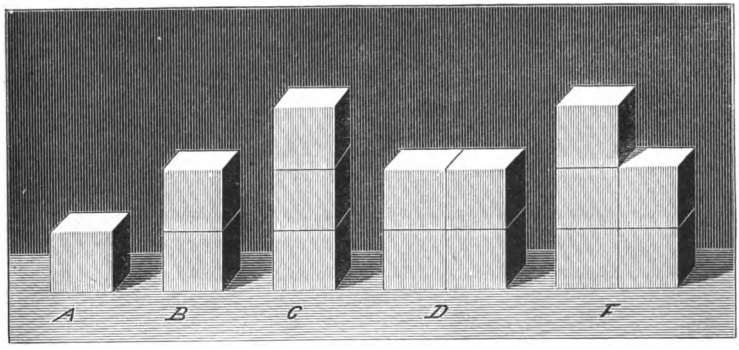
Do you go to church twice on Sunday?

How many times does the clock strike when it is 4 o'clock?

How many times have you been late this term?

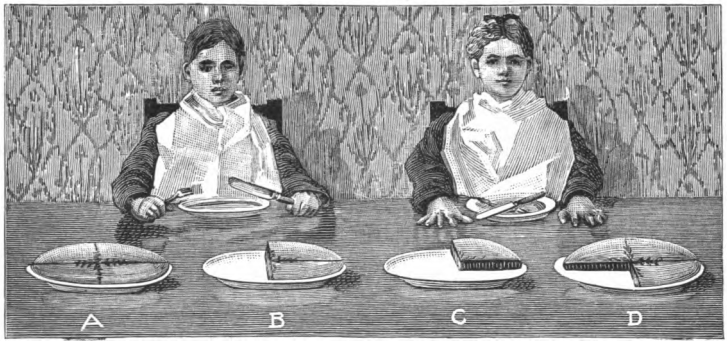
How many days in the week do you go to school?

How Many Times as Large



How many times does B contain A?
What is the relation of B to A? *Ans.* B is 2 times A.
How many times does D contain B?
D is how many times as large as B?
What is the relation of D to B? *Ans.* D is 2 times B.
How many times does C contain A?
C is how many times as large as A?
What is the relation of C to A?
How many times does F contain A?
F is how many times as large as A?
What is the relation of F to A?
D is how much larger than A?
D is how many times as large as A?
What block is two times as large as B?
What block is twice as large as A?
B is how much larger than A? *Ans.* 1 cube.
What is the relation of B to A?
Ans. B is one cube larger than A.
State the relation of D to A in two ways.

What Part



What part of the whole pie is B? *Ans.* One half.

How is one half written? *Ans.* $\frac{1}{2}$.

What part of the whole pie is C? *Ans.* One fourth.

Is one fourth ever called anything else?

Ans. Yes; one quarter.

How is one quarter written? *Ans.* $\frac{1}{4}$.

How is one fourth written? *Ans.* $\frac{1}{4}$.

Which is the larger, half a pie or quarter of a pie?

C is what part of B? A quarter of a pie is what part of half a pie?

How many of B make A? How many halves make a whole?

How many of C make A? How many quarters make a whole?

How much of a pie is C? *Ans.* A quarter.

How many pieces like C will make D?

How much does D lack of being a whole pie?

D is how many times C? How much of the pie is D?

Ans. Three quarters.

How is three quarters written? *Ans.* $\frac{3}{4}$.

What Order

Counting from the front, which is the pear? *Ans.* The first.

Which are the cherries? *Ans.* The second.

Which is the banana? The apple? The pineapple?

Name the first from the back; the second; the third; the fourth; the fifth.

Which one is in the middle? Which one is the first in the rear of the middle?

Which is the second in the rear of the middle?

Which is the first in front of the middle? The second?

Who sits the first on your right?

Who sits the farthest from you on your right?

Who sits the fifth on your left? The third on your right? The first in front of you? The second behind you?

Counting from yourself, name the four little friends who sit in front of you.

Write the words which tell the order in which they sit.

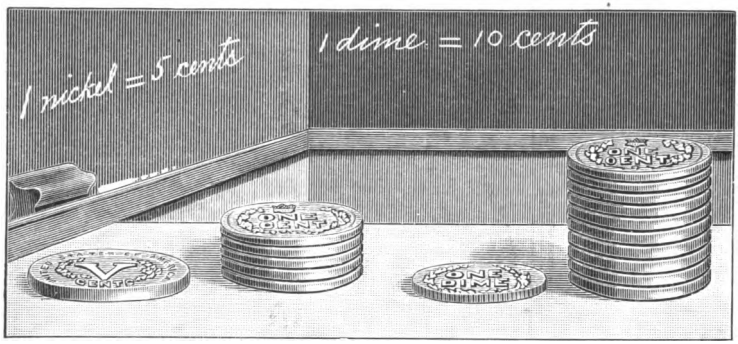
Did you spell the word right the first time?

Did you catch the ball the second time?

Can you tell me the name of the first day of the week?

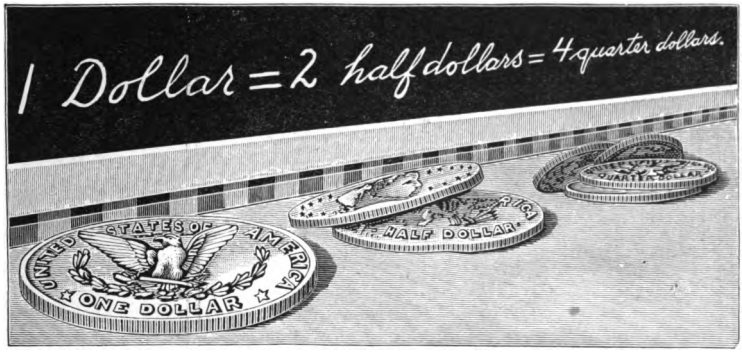
NOTATION AND NUMERATION

How Many — To Ten



1. Take a splint and say *one*, 1.
2. Take another and say *two*, 2.
3. Take another and say *three*, 3.
4. Take another and say *four*, 4.
5. Take another and say *five*, 5.
6. Take another and say *six*, 6.
7. Take another and say *seven*, 7.
8. Take another and say *eight*, 8.
9. Take another and say *nine*, 9.
10. Take another and say *ten*, 10.
11. Lay these ten splints down, pick up one of them and say, "This is the first; I have one."
12. Pick up another and say, "This is the second; I have two."
13. Pick up another and say, "This is the third; I have three."
14. Continue in this way until you have picked up all the splints.

How Many — To Ten



Let us play store.

A nickel will buy as much as 5 pennies.

15. You may pay me for five cents' worth of bananas; count out 5 pennies.

16. You may pay me for a 5-cent orange; this time you may give me a nickel.

17. I will sell you a top for 6 cents; count out 6 pennies.

18. Instead of 6 pennies, you may hand me a nickel and say 5, then a penny and say 6.

19. With pennies you may pay me 7 cents for a doll.

20. You may pay for the doll with a nickel and pennies.

21. Please give me in pennies 8 cents for a spool of thread.

22. You may pay me with a nickel and pennies.

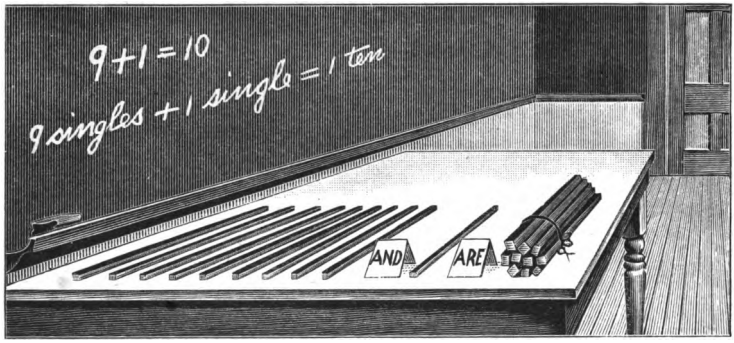
23. I will sell you a knife for 9¢; please pay me in pennies.

24. You may pay me with a nickel and pennies.

25. You may pay me in any way you please for a seven-cent top.

26. You may pay me 9 cents for 8 marbles.

How Many — Ten to Twenty



27. Take nine splints; take one more; bind them together; you will have a bundle of ten.

28. Take one more splint and say eleven. What does eleven mean? *Ans.* 1 ten 1.

29. Take one more splint and say twelve. What does twelve mean? *Ans.* 1 ten 2.

30. Take one more splint and say thirteen. What does thirteen mean? *Ans.* 1 ten 3.

NOTE.—The teacher should continue in the same way to twenty.

31. Take ten splints (1 bundle of ten) and write 10. What does 10 mean? *Ans.* 1 ten 0.

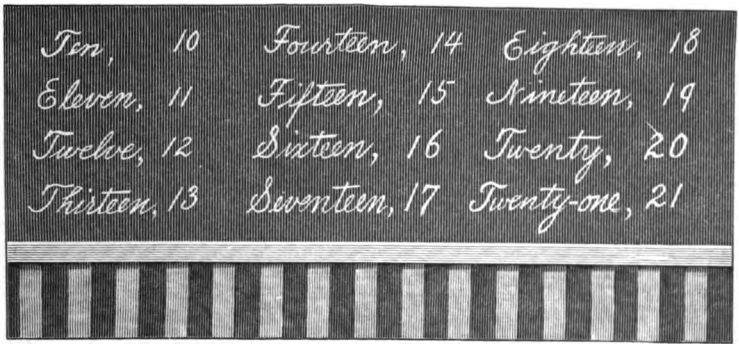
32. Take eleven splints (1 bundle of ten and 1) and write 11. What does 11 mean? *Ans.* 1 ten 1.

33. Take twelve splints (1 bundle of ten and 2) and write 12. What does 12 mean? *Ans.* 1 ten 2.

34. Take 13 splints (1 bundle of ten and 3) and write 13. What does 13 mean? *Ans.* 1 ten 3.

NOTE.—The teacher should continue in the same way to 20.

How Many — Ten to Twenty



Let us play store.

A dime will buy as much as 10 pennies. *See p. 20.*

35. I will sell you a ball for 10 cents; you may pay me with pennies.

36. You may pay for the ball with a nickel and pennies.

37. You may pay for the ball with a dime.

38. A pound of cheese will cost you 14 cents. You may pay with a dime and pennies; hand me a dime and say ten, a penny and say 11; and so on.

39. With a dime and pennies you may pay me 19 cents for a basket.

40. How much is a dime and a nickel?

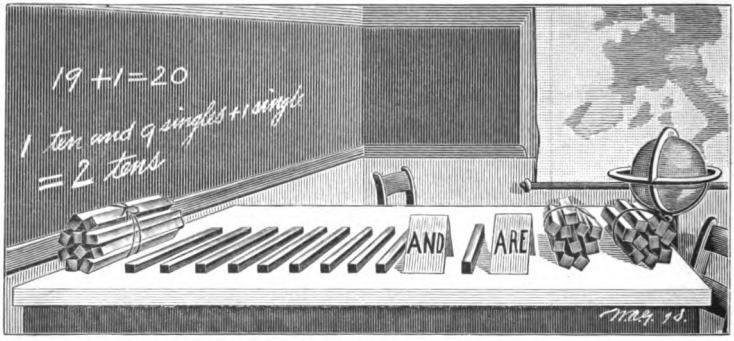
Ans. 1 ten 5, or 15 cents.

41. You may pay me for the basket with a dime, a nickel, and pennies. Hand me a dime and say 10, a nickel and say 15, a penny and say 16, and so on.

42. With pennies you may pay me 18 cents for a yard of ribbon.

43. You may pay for the ribbon with a dime and pennies.

How Many — Twenty to One Hundred



44. Take 19 splints (1 bundle of 10 and 9); take 1 more; bind together the ten single splints; you will have 2 bundles of ten, or twenty.

45. What does twenty mean? *Ans.* 2 ten 0.

46. How is twenty written? *Ans.* 20.

47. Take up one splint at a time and say twenty-one, twenty-two, twenty-three, twenty-four, twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine.

48. Take one more splint; bind together the ten single splints; you will have three bundles of ten, or 30.

49. Without splints, name the numbers: from 30 to 40; from 40 to 50; from 50 to 60.

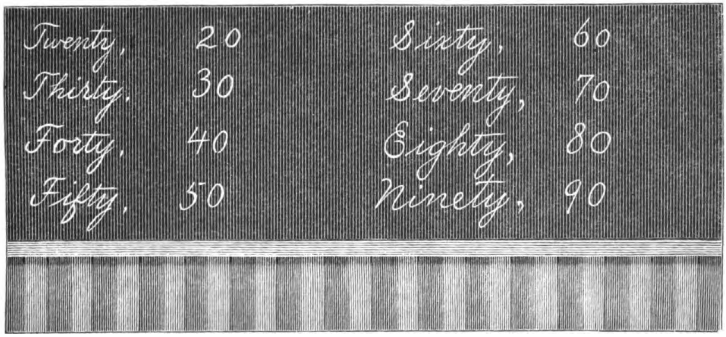
50. Without splints, name the numbers: from 60 to 70; from 70 to 80; from 80 to 90.

51. Take 99 splints (9 bundles of ten and 9); take 1 more; bind together the ten single splints; bind together the ten 'tens'; you will have a bundle of 'one hundred.'

52. What does one hundred mean? *Ans.* 1 hundred 0 ten 0.

53. How is one hundred written? *Ans.* 100.

How Many — Twenty to One Hundred



Let us play store. *See p. 21.*

A half dollar equals 50 pennies; a quarter, 25 pennies.

54. I will sell you a melon for 38 cents; you may pay me with pennies.

55. You may pay me with a quarter and pennies; hand me a quarter and say 25, a penny and say 26, and so on.

56. You may pay me with a quarter, a dime, and pennies; hand me a quarter and say 25, a dime and say 35, a penny and say 36, and so on.

57. With pennies, you may pay me 69 cents for a book.

58. You may pay me for the book with a half dollar and pennies; hand me a half dollar and say 50, a penny and say 51, and so on.

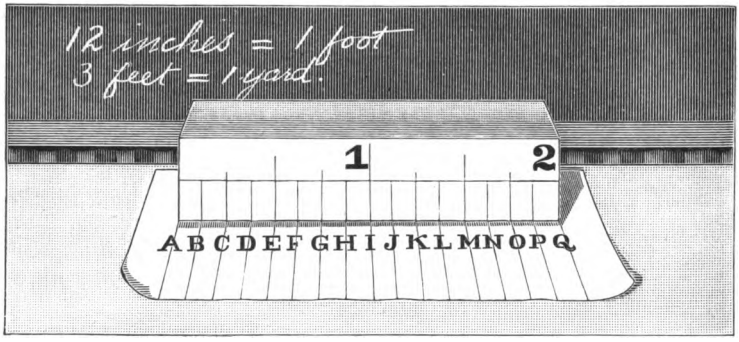
59. You may pay me for the book with a half dollar, a dime, and pennies; hand me a half dollar and say 50, a dime and say 60, a penny and say 61, and so on.

60. You may pay me for the book in some other way.

61. You may count out 98 pennies to pay for a lamp.

62. You may pay me 79 cents for a hat.

Measurements — Lines



63. Above is a two-inch rule. What is the length from A to E? *Ans.* Half an inch.

64. What is the length from A to C? From A to B?

65. How many AB's make AC? How many eighths make a quarter?

66. How many AB's make AE? How many eighths make a half?

67. How many AB's make AI? How many eighths make an inch?

68. How many AC's make AE? How many quarters make a half?

69. How many AC's make AI? How many quarters make an inch?

70. Draw on paper a one-inch rule and cut it out. Be sure to make it exactly one inch, and divide it as above.

71. Draw a line whose length is 1 inch; 1 quarter; 1 half.

72. Draw a line whose length is 3 inches; 3 quarters; 3 halves.

Measurements — Lines

73. Draw a line whose length is 1 inch and 1 half; how many half inches?

74. Draw a line whose length is 1 half and 1 quarter; how many quarters?

75. Draw a line whose length is 1 quarter and 1 eighth; how many eighths?

76. Draw a line whose length is 1 half, 1 quarter, and 1 eighth; how many eighths?

77. Measure the length of this book; its width; its thickness.

78. Is your one-inch rule convenient for finding the length of this room?

—•—

79. Cut out a strip of paper one inch wide and 12 inches long; at the end of the first inch draw a line from the edge and write 1; at the end of the next inch draw a line and write 2; and so on to 12. Divide each inch space as in the drawing. You will have a foot rule.

80. Measure the height of some little boy.

81. Measure the width of this room; its length.

82. How far do you guess it to be from the floor to the ceiling?

—•—

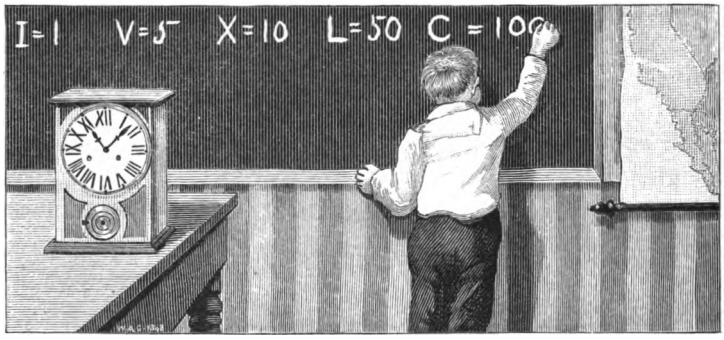
83. Paste strips of paper together and make a rule 3 feet long. You will have a yard rule.

84. In what store have you seen a yard rule used?

85. Is your yard rule convenient for finding the distance to the post office?

Ans. No. A chain 66 feet long would be used.

Roman Notation



86. Count to five in this way; one, two, three, one from five, five.

87. Count as before, and write the numbers by letters.

Ans. One, I; two, II; three, III; one from five, IV; five, V.

88. Look at the clock face and see how the numbers from one to five are written.

89. Why is 'four' written IIII on the clock instead of IV?

Ans. IIII was written at first by mistake, and no change has since been made.

90. Count from five to ten; five, five and one, five and two, five and three, one from ten, ten; write the numbers by letters; compare with the clock face.

Ans. Five, V; five and one, VI; five and two, VII; five and three, VIII; one from ten, IX; ten, X.

91. Count and write from ten to twenty; see the clock.

Ans. Ten, X; eleven (ten and one), XI; twelve (ten and two), XII; thirteen (ten and three), XIII; fourteen (ten and four), XIV; fifteen (ten and five), XV; sixteen (ten and six), XVI; seventeen (ten and seven), XVII; and so on.

ADDITION

The 45 Combinations

$2, \overset{1}{1}.$

1. Take 2 cubes; place each by itself; put them together again.

2. How many are 1 and 1? Find '?': $1 + ? = 2$.

3. Name two numbers whose sum is two.

Ans. One one. Write 2. *Ans.* $\overset{1}{1}$. Read $\overset{1}{1}$. *Ans.* 2.

4. Draw a domino and put 2 spots on one side.

5. Draw a domino and put 1 spot on each side.

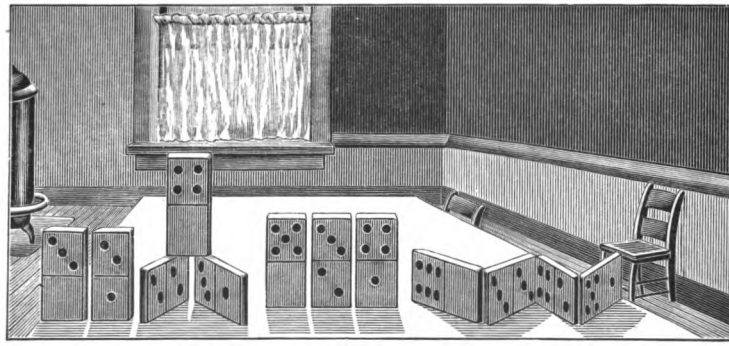
6. A boy has 1 apple in each hand. How many apples has he?

7. A girl has 1 doll in her hand and 1 doll on the floor. Ask a question and answer it. *Ans.* How many dolls has she? 2.

8. Tell a story about "$1 + 1 = 2$."

Ans. Mary had 1 pin, and her aunt gave her 1 more. She then had 2 pins.

The 45 Combinations



$3, \overset{2}{1}.$

9. Take 3 cubes; make 2 piles; how many in each pile?

10. Place the 3 cubes in one pile again; how many are 2 and 1?

11. Name two numbers whose sum is 3. *Ans.* Two one.

12. Find '?': $1+?=3$; $2+?=4$; $?+3=4$; $3+?=4$.

13. Write 3. *Ans.* $\overset{2}{1}$. Read $\overset{2}{1}$. *Ans.* 3.

14. Point out the dominoes in the picture that show 3.

15. If John had 2 cents and earned 1 cent, how much money had he then?

16. James had 1 top in his hand and 2 tops on the floor. Ask a question and answer it.

17. Tell a story about "2 + 1 = 3."

18. Draw a line a foot long; measure off an inch; measure off 2 inches more. How much have you measured off in all?

19. Mary bought 1 yard of cloth and then 2 yards more. Ask a question and answer it.

The 45 Combinations

4, $\overset{2}{2},\ \overset{3}{1}$.

20. Take 4 cubes; make two piles with the same number in each; how many in each pile? How many are 2 and 2?

21. Take 1 cube from the smaller pile and place it on the other; how many in each pile? How many are 3 and 1?

22. Name all the pairs of numbers whose sum is 4.

Ans. Two two, three one.

23. Write 4 in two ways. *Ans.* $\overset{2\ 3}{2, 1}$. Read $\overset{2\ 3}{2, 1}$. *Ans.* 4, 4.

24. Draw dominoes as in the picture to show 4.

25. Mary has drawn a line 2 inches long and Susan has drawn a line of the same length. Ask a question and answer it.

26. Tell a story about "1 + 3 = 4."

5, $\overset{3}{2},\ \overset{4}{1}$.

27. Take 5 splints; make two piles with as nearly the same number in each as possible; how many are in each pile? How many are 3 and 2?

28. Take 1 splint from the smaller pile and place it on the other; how many in each pile? How many are 4 and 1?

29. Name pairs whose sum is 5. *Ans.* Three two, four one.

30. Draw dominoes as in the picture to show 5.

31. Read: $\overset{3}{2},\ \overset{4}{1};\ \overset{2}{1},\ \overset{2}{2},\ \overset{3}{1}$. *Ans.* 5, 5; 3, ...

32. Five is an *odd* number, because with 5 splints we cannot make two piles that shall have the same number in each. Is 4 an odd number?

The 45 Combinations

6, $\overset{3}{3},\ \overset{4}{2},\ \overset{5}{1}$.

33. Take 6 splints; make two piles with the same number in each; how many in each pile? How many are 3 and 3?

34. Take 1 splint from one pile and place it on the other; how many in each pile? How many are 4 and 2?

35. Take another splint from the smaller and place it on the larger; how many in each pile? How many are 5 and 1?

36. Name the pairs whose sum is 6.

Ans. Three three, four two, five one.

37. Write 6 in three ways.

38. Find '?': $2 + ? = 6$; $3 + ? = 6$; $? + 4 = 6$.

	3	4	5	2	1	2	3	4

39. Read: 3, 2, 1; 2, 1, 1, 1, 1. *Ans.* 6, 6, 6; 4 ···

40. 6 is an *even* number, because with 6 splints we can make two piles that shall have the same number in each. Is 3 an even number?

NOTE. — The teacher should supply concrete problems.

7, $\overset{4}{3},\ \overset{5}{2},\ \overset{6}{1}$.

41. Take 7 splints; make two piles with as nearly the same number in each as possible; and proceed as in developing 6.

42. Name the pairs whose sum is 7.

Ans. Four three, five two, six one.

43. Name pairs for 7, 5, 3, 4, 2, 6.

	4	5	6	3	4	5	3	4	2	3	2	1

44. Read: 3, 2, 1; 3, 2, 1, 2, 1, 2, 1, 1, 1.

45. Draw dominoes in three ways to show 7.

The 45 Combinations

8, $\begin{matrix}4\\4,\end{matrix}$ $\begin{matrix}5\\3,\end{matrix}$ $\begin{matrix}6\\2,\end{matrix}$ $\begin{matrix}7\\1.\end{matrix}$

46. Take 8 splints ; make two piles with as nearly the same number in each as possible ; how many in each pile? How many are 4 and 4?

47. Take 1 splint from the smaller pile and place it on the other ; how many in each pile? How many are 5 and 3?

48. Take 1 splint from the smaller pile and place it on the other ; how many in each pile? How many are 6 and 2?

49. Take 1 splint from the smaller pile and place it on the other ; how many in each pile? How many are 7 and 1?

50. Name the pairs whose sum is 8.

	4	5	6	7	4	3	3	2	2	1	3	4
51. Read :	4,	3,	2,	1;	3,	3,	2,	2,	1,	1,	1,	1.

52. Find '?': $3+?=8$; $6+?=8$; $5+?=8$; $?+2=8$.

9, $\begin{matrix}5\\4,\end{matrix}$ $\begin{matrix}6\\3,\end{matrix}$ $\begin{matrix}7\\2,\end{matrix}$ $\begin{matrix}8\\1.\end{matrix}$

53. Take 9 splints, and, proceeding as before, name the pairs for 9.

	5	6	7	8	7	5	4	5	6	4	3	4
54. Read :	4,	3,	2,	1;	1,	3,	3,	2,	2,	2,	3,	4.

10, $\begin{matrix}5\\5,\end{matrix}$ $\begin{matrix}6\\4,\end{matrix}$ $\begin{matrix}7\\3,\end{matrix}$ $\begin{matrix}8\\2,\end{matrix}$ $\begin{matrix}9\\1.\end{matrix}$

55. Take 10 splints and name the pairs for 10.

56. Name the pairs for: 9; 6; 3; 5; 4; 7; 8.

	5	6	7	8	9	7	4	6	7	5	8	8
57. Read :	5,	4,	3,	2,	1;	1,	4,	4,	2,	4,	2,	1.

NOTE. — The teacher should continue such exercises as in Ex. 52.

The 45 Combinations

11, $\overset{6}{5},\ \overset{7}{4},\ \overset{8}{3},\ \overset{9}{2}.$

58. Take 11 splints and name the pairs for 11. Stop with "nine two" because 9 is the largest number that can be written with one figure.

59. Write 11 in four ways. Read: 6 5, 7 4, 8 3, 9 2.

60. A man has two boards, one 6 ft. long and one 5 ft. long. What is their combined length?

12, $\overset{6}{6},\ \overset{7}{5},\ \overset{8}{4},\ \overset{9}{3}.$

61. Take 12 splints and name the pairs for 12. Stop with "nine three."

62. Write 12 in four ways.

63. Read: 6 6, 7 5, 8 4, 9 3; 6 5, 3 2, 8 1, 9 1, 7 3, 8 2, 4 4, 5 5, 6 6, 3 3.

13, $\overset{7}{6},\ \overset{8}{5},\ \overset{9}{4}.$

64. Take 13 splints and name the pairs for 13. Stop with "nine four."

65. Write 13 in three ways. Read: 7 6, 8 5, 9 4.

66. Tell a story about "7 + 6 = 13."

14, $\overset{7}{7},\ \overset{8}{6},\ \overset{9}{5}.$

67. Take 14 splints and name the pairs for 14. Stop with "nine five."

68. Write 14 in three ways. Read: 7 7, 8 6, 9 5.

69. Mary has 8 ¢ and Jane has 6 ¢. Ask a question.

The 45 Combinations

15, $^{8}_{7}$, $^{9}_{6}$.

70. Take 15 splints and name the pairs for 15. Stop with "nine six."

71. Write 15 in two ways. Read:

8	9	7	8	6	9
7,	6;	5,	3,	2,	5.

72. How many are 6 apples and 9 apples?

16, $^{8}_{8}$, $^{9}_{7}$.

73. Take 16 splints and name the pairs for 16. Stop with "nine seven."

74. Write 16 in two ways. Read:

8	9	6	7	9	6
8,	7;	3,	4,	2,	5.

75. What is the value of $8 + 8$?

17, $^{9}_{8}$.

76. Take 17 splints and name a pair for 17.

77. Write 17 in one way. Read:

9
8.

78. $9 + 8 =$ what?

18, $^{9}_{9}$.

79. Take 18 splints and name a pair for 18.

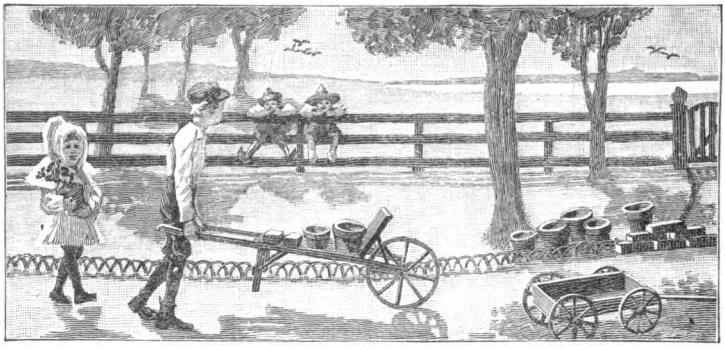
80. Write 18 in one way. Read:

9
9.

81. How many pairs make: 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14? 15? 16? 17? 18?

82. How many combinations are there of the first nine numbers taken two and two?

The 45 Combinations



83. Make and answer an example about the flower pots.

84. Make and answer an example about the trees.

85. Make and answer an example about the wheels.

86. Two little maidens going for a walk took six other little maidens. How many maidens went walking?

87. My plant is 2 inches high. If it grows 3 inches more, how high will it be?

88. There are 8 balls in the basket and 2 in my hand. How many balls have I altogether?

89. Anna used 4 buttons on her doll's cape and 5 on the dress. How many did she use altogether?

90. How many feet have Harry and his dog Towser?

91. How many cents do you need to buy a 3-cent pear and a 5-cent peach?

92. Carrie's aunt gave her all the change she had in her pocketbook, a 5-cent piece and 6 pennies. How much did she give her?

93. Fred's mamma gave him 9 cents and 6 cents. How much did she give him in all?

The 45 Combinations

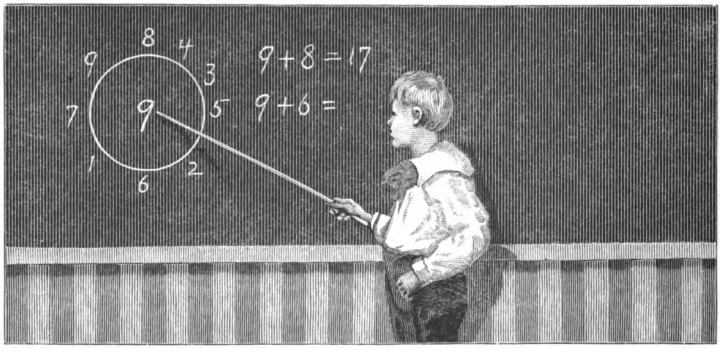
Call the sums rapidly:

	1	8	3	6	5	8		4	6	7	4	9	3
94.	9,	2,	7,	4,	9,	1	**100.**	9,	7,	3,	2,	1,	6
	4	6	3	8	4	1		7	8	2	4	5	3
95.	1,	2,	9,	5,	2,	7	**101.**	1,	8,	2,	2,	9,	5
	7	3	4	5	9	3		5	4	9	7	3	2
96.	2,	2,	6,	1,	2,	7	**102.**	2,	1,	6,	5,	1,	9
	8	7	2	8	6	4		1	4	7	3	6	2
97.	4,	1,	9,	5,	3,	7	**103.**	5,	8,	2,	9,	1,	5
	1	5	6	9	2	7		7	3	5	2	4	7
98.	8,	2,	8,	9,	8,	4	**104.**	5,	3,	3,	6,	9,	6
	5	9	8	7	6	5		7	1	6	1	9	7
99.	6,	2,	4,	3,	9,	1	**105.**	2,	9,	3,	1,	7,	1

Call the sums rapidly:

	5	7	4	6	9	3		3	7	5	1	9	6
106.	3,	1,	5,	2,	1,	8	**112.**	8,	4,	2,	4,	3,	8
	4	6	3	1	6	5		6	4	5	7	8	5
107.	5,	9,	8,	1,	4,	6	**113.**	3,	5,	5,	2,	1,	8
	9	7	9	4	8	4		8	7	4	6	9	8
108.	2,	8,	7,	4,	1,	8	**114.**	8,	6,	3,	6,	4,	6
	2	6	8	3	5	9		6	9	3	1	7	2
109.	3,	6,	4,	5,	6,	7	**115.**	5,	8,	6,	3,	6,	7
	9	8	7	9	5	4		7	6	8	9	8	3
110.	8,	6,	5,	6,	6,	7	**116.**	3,	1,	2,	9,	5,	9
	1	9	6	2	3	5		8	9	6	2	7	1
111.	6,	8,	5,	6,	3,	8	**117.**	6,	4,	7,	1,	6,	8

The 45 Combinations



118. Point to 9 and 8 and declare their sum. In the same way treat: 9 and 4; 9 and 3; and so on around.

119. Place 8 in the center of the circle; declare the sums as before.

120. Continue the exercise by placing in the center: 7; 6; 5; 4; 3, 2; 1.

Copy and supply the number for '?':

121.	$9+8=?$	$7+6=?$	$8+8=?$
122.	$6+9=?$	$5+4=?$	$7+7=?$
123.	$7+8=?$	$9+7=?$	$6+6=?$
124.	$5+6=?$	$4+4=?$	$9+9=?$
125.	$8+3=?$	$5+7=?$	$8+4=?$

Copy and supply the number for '?':

126.	$8+6=?$	$3+3=?$	$4+3=?$
127.	$7+5=?$	$2+2=?$	$6+3=?$
128.	$6+4=?$	$7+4=?$	$2+1=?$
129.	$9+4=?$	$5+5=?$	$8+4=?$
130.	$7+6=?$	$9+7=?$	$4+4=?$

The 45 Combinations

Ask a question and give the answer up to Ex. 143:

131. In an orchard there are 8 peach-trees and 6 apple-trees. *Ans.* How many trees are there in the orchard? 14 trees.

132. A house has 8 rooms upstairs and 6 downstairs.

133. Fred earned 8 cents on Monday and 9 cents on Tuesday.

134. Tommy broke his fish pole into two pieces; one piece was 8 ft. long, and the other 7 ft. long.

135. The head of a fish is 4 inches long; the body and tail are 9 inches long.

136. A boy rode 7 miles and back on his bicycle.

137. Frank saw two flocks of prairie chickens; in the first were 7 chickens; in the second, 4.

138. Mary has 7 paper dolls; Kate has 6 more than Mary.

139. James has a ribbon 9 inches long. It is 5 inches shorter than Mary's.

140. A bunch of violets will cost me 3 cents, a cluster of roses 9 cents.

141. Charles bought oranges for 6 cents, and candy for 9 cents.

142. A man bought a coat for 9 dollars and a vest for 5 dollars.

143. Make an example about "2 cents + 8 cents," and give the answer.

144. Make an example about "9 dogs + 8 dogs," and give the answer.

145. Make an example about "6 windows + 3 windows," and give the answer.

146. Make an example about "3 oranges + 7 oranges," and give the answer.

Counting by the First Ten Numbers

147. Take 12 splints (1 bundle of ten and 2) and 5 splints. How many have you? *Ans.* 1 bundle of ten and 7, or 17.

148. Count from 1 to 19 by 2's.

Ans. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19.

Count from:

149. 2 to 18 by 2's

150. 1 to 19 by 3's

151. 2 to 17 by 3's

152. 3 to 18 by 3's

153. 1 to 17 by 4's

154. 2 to 18 by 4's

155. 3 to 19 by 4's

156. 4 to 16 by 4's

157. 1 to 16 by 5's

158. 2 to 17 by 5's

159. 3 to 18 by 5's

160. 4 to 19 by 5's

161. 5 to 15 by 5's

162. 1 to 19 by 6's

Count from:

163. 2 to 14 by 6's

164. 3 to 15 by 6's

165. 4 to 16 by 6's

166. 5 to 17 by 6's

167. 6 to 18 by 6's

168. 1 to 15 by 7's

169. 2 to 16 by 7's

170. 3 to 17 by 7's

171. 4 to 18 by 7's

172. 5 to 19 by 7's

173. 1 to 17 by 8's

174. 2 to 18 by 8's

175. 3 to 19 by 8's

176. 1 to 19 by 9's

177. Count from 1 to 91 by 10's.

Ans. 1, 11, 21, 31, 41, 51, 61, 71, 81, 91.

178. Count from: 2 to 92 by 10's; from 3 to 93; from 4 to 94; from 5 to 95.

Counting by the First Ten Numbers

179. Take 15 splints (1 bundle of ten and 5) and 9 splints; put the single splints together, making 1 bundle of ten and 4. How many have you? *Ans.* 2 bundles of ten and 4, or 24.

180. Take 28 splints (2 bundles of ten and 8) and 7 splints; put the single splints together, making 1 bundle of ten and 5. How many have you?

How many are:

181. $15+9$? $25+9$? $35+9$? $45+9$? $55+9$? $65+9$? $75+9$? $85+9$?

182. $28+7$? $38+7$? $48+7$? $58+7$? $68+7$? $78+7$? $88+7$? $18+7$?

183. $34+6$? $54+6$? $84+6$? $44+6$? $24+6$? $64+6$? $74+6$? $14+6$?

How many are:

184. $28+8$? $88+8$? $68+8$? $38+8$? $48+8$? $18+8$? $58+8$? $78+8$?

185. $19+9$? $89+9$? $79+9$? $49+9$? $29+9$? $69+9$? $39+9$? $59+9$?

186. $35+7$? $45+7$? $55+7$? $65+7$? $75+7$? $85+7$? $25+7$? $15+7$?

How many are:

187. $73+2$? $84+3$? $18+6$? $29+7$? $33+8$? $42+9$? $55+6$? $67+2$?

188. $55+6$? $68+3$? $18+7$? $83+6$? $82+8$? $85+9$? $63+8$? $74+6$?

189. $63+7$? $48+9$? $56+8$? $78+6$? $94+3$? $88+9$? $75+5$? $83+8$?

SUBTRACTION

The 45 Combinations

$2, \overset{1}{1}.$

1. Name two numbers whose sum is 2. *Ans.* **One one.** Write $2. \overset{1}{1}.$ *Ans.* 1.

2. To make 2, how many must be added to 1? *Ans.* **1.**

3. How many are $2 - 1$? *Ans.* 1.

4. Lulu had a pair of white rabbits, but one ran away. How many has she now?

5. Anna wants a ruler that costs 2 cents, but she has only 1 cent. Ask a question. *Ans.* How many cents must she get?

6. Tell a story about "$2 - 1 = 1$."

NOTE. — The forty-five combinations in subtraction may be learned in a very short time after the same combinations in addition have been mastered. After p. 29, the teacher may prefer to take up this page.

The 45 Combinations

3, $\begin{smallmatrix}2\\1\end{smallmatrix}$.

7. Name two numbers whose sum is 3. *Ans.* Two one. Write 3. *Ans.* $\begin{smallmatrix}2\\1\end{smallmatrix}$.

8. To make 3, how many must be added to 1? to 2?

9. How many are: $3-1$? $3-2$?

10. John has 3 walnuts. If he gives 1 to Mabel, how many will he have?

11. Topknot had 3 little chicks; a hawk carried 2 away. Ask a question.

12. Tell a story about "$3-1=2$."

13. Tell a story about "$3-2=1$."

4, $\begin{smallmatrix}2\\2\end{smallmatrix}$, $\begin{smallmatrix}3\\1\end{smallmatrix}$.

14. Name all the pairs of numbers that make 4.

15. To make 4, how many must be added to 1? to 2? to 3?

16. How many are: $4-2$? $4-3$? $4-1$?

17. Lucy had 4 cents; she spent 3 cents. How many cents had she then?

18. Frank has 4 dollars; his brother has 2 dollars. Ask a question.

19. Make an example about "$4-2=2$."

20. Make an example about "$4-1=3$."

21. Make an example about "$4-3=1$."

NOTE. — This book would be too bulky if all the questions were asked which may be necessary. The above are only suggestive. In the other combinations, problems have been omitted. They should be supplied by the teacher.

The 45 Combinations

5, $\begin{smallmatrix}3&4\\2,&1.\end{smallmatrix}$

22. Name the pairs for 5. Write 5 in two ways.

23. To make 5, how many must be added to 1? to 2? to 3? to 4?

24. How many are: 5 – 2? 5 – 4? 5 – 3? 5 – 1?

6, $\begin{smallmatrix}3&4&5\\3,&2,&1.\end{smallmatrix}$

25. Name the pairs for 6. Write 6 in three ways.

26. To make 6, how many must be added to 1? to 2? to 3? to 4? to 5?

27. How many are: 6 – 3? 6 – 2? 6 – 4? 6 – 1? 6 – 5?

7, $\begin{smallmatrix}4&5&6\\3,&2,&1.\end{smallmatrix}$

28. Name the pairs for 7. Write 7 in three ways.

29. To make 7, how many must be added to 1? to 2? to 3? to 4? to 5? to 6?

30. How many are: 7 – 5? 7 – 3? 7 – 4? 7 – 1? 7 – 2? 7 – 6?

8, $\begin{smallmatrix}4&5&6&7\\4,&3,&2,&1.\end{smallmatrix}$

31. Name the pairs for 8. Write 8 in four ways.

32. How many are: 8 – 2? 8 – 3? 8 – 1? 8 – 7? 8 – 5? 8 – 6? 8 – 4?

9, $\begin{smallmatrix}5&6&7&8\\4,&3,&2,&1.\end{smallmatrix}$

33. Name the pairs for 9. Write 9 in four ways.

34. How many are: 9 – 3? 9 – 6? 9 – 2? 9 – 8? 9 – 7? 9 – 1? 9 – 4? 9 – 5?

The 45 Combinations

10, $\overset{5}{5},\ \overset{6}{4},\ \overset{7}{3},\ \overset{8}{2},\ \overset{9}{1}.$

35. Name the pairs for 10. Write 10 in five ways.

36. How many are: $10-2$? $10-4$? $10-6$? $10-8$? $10-9$? $10-7$? $10-5$? $10-3$? $10-1$?

11, $\overset{6}{5},\ \overset{7}{4},\ \overset{8}{3},\ \overset{9}{2}.$

37. Name the pairs for 11. Write 11 in four ways.

38. How many are: $11-3$? $11-8$? $11-6$? $11-2$? $11-4$? $11-7$? $11-5$? $11-9$?

12, $\overset{6}{6},\ \overset{7}{5},\ \overset{8}{4},\ \overset{9}{3}.$

39. Name the pairs for 12. Write 12 in four ways.

40. How many are: $12-3$? $12-8$? $12-6$? $12-4$? $12-7$? $12-5$? $12-9$?

13, $\overset{7}{6},\ \overset{8}{5},\ \overset{9}{4}.$

41. Name the pairs for 13. Write 13 in three ways.

42. How many are: $13-8$? $13-5$? $13-9$? $13-7$? $13-4$? $13-6$?

14, $\overset{7}{7},\ \overset{8}{6},\ \overset{9}{5};$ **15,** $\overset{8}{7},\ \overset{9}{6}.$

43. Treat 14 like preceding numbers. Treat 15 in the same way.

16, $\overset{8}{8},\ \overset{9}{7};$ **17,** $\overset{9}{8};$ **18,** $\overset{9}{9}.$

44. Treat 16 in the same way. Treat 17 in the same way. Treat 18 in the same way.

The 45 Combinations

Declare the remainders:

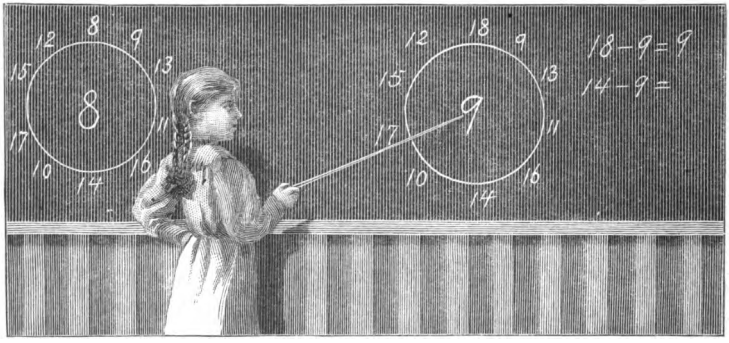
	8	6	3	7	5	8	9	4	7
45.	7	2	1	4	3	5	6	1	3
	2	8	9	5	7	9	3	4	8
46.	1	3	2	1	6	8	3	2	5
	6	9	7	8	3	6	8	7	9
47.	4	5	4	2	2	1	4	5	2
	8	9	6	7	9	6	4	5	8
48.	4	6	3	2	8	5	3	4	1
	5	6	5	9	8	7	9	3	4
49.	2	5	3	7	1	7	4	2	4
	5	6	8	7	9	8	2	9	3
50.	5	6	8	1	9	6	2	1	2
	4	1	8	9	3	2	9	8	7
51.	2	1	6	8	1	1	3	3	6
	5	8	9	4	7	5	6	7	8
52.	2	4	5	3	5	3	4	2	7
	7	8	9	7	9	5	6	8	4
53.	3	5	7	1	3	4	2	1	1

The 45 Combinations

Declare the remainders:

	13	12	14	17	11	12	16	11	10
54.	5	8	7	8	4	5	7	5	1
	15	16	14	10	15	16	14	13	12
55.	7	9	6	3	9	8	5	7	9
	11	15	14	12	13	17	10	12	13
56.	6	8	9	3	6	9	2	7	8
	14	12	15	10	13	14	18	15	10
57.	9	6	6	5	4	8	9	7	6
	13	11	10	12	17	13	11	16	11
58.	5	4	8	7	8	9	9	7	6
	14	16	11	10	13	17	11	10	18
59.	8	9	5	6	4	9	7	4	9
	13	12	15	11	13	10	14	12	16
60.	6	5	8	2	7	9	5	3	8
	10	14	11	14	12	16	12	17	13
61.	6	7	9	9	8	7	4	8	9
	11	15	14	13	12	11	10	14	13
62.	3	8	7	5	6	2	7	6	8

The 45 Combinations



63. Point to 18 and 9 and declare their difference. In the same way: treat 9 and 9; 13 and 9; and so on.

64. Place 8 in the center, and in place of the largest number on the outside (18); declare the difference as before.

65. Continue by placing in the center and for the largest number left on the outside: 7; 6; 5; 4; 3; 2; 1.

Copy and supply the number for '?':

66.	$7 + ? = 16$	$8 + ? = 16$	$9 + ? = 12$
67.	$8 + ? = 17$	$7 + ? = 15$	$8 + ? = 10$
68.	$3 + ? = 10$	$9 + ? = 18$	$8 + ? = 15$
69.	$9 + ? = 13$	$3 + ? = 11$	$7 + ? = 11$
70.	$6 + ? = 12$	$9 + ? = 17$	$8 + ? = 16$

Copy and supply the number for '?':

71.	$15 - 7 = ?$	$16 - 8 = ?$	$18 - 9 = ?$
72.	$11 - 6 = ?$	$13 - 4 = ?$	$15 - 6 = ?$
73.	$10 - 1 = ?$	$12 - 7 = ?$	$12 - 3 = ?$
74.	$12 - 4 = ?$	$11 - 2 = ?$	$14 - 5 = ?$
75.	$14 - 7 = ?$	$10 - 4 = ?$	$11 - 4 = ?$

The 45 Combinations

Ask a question and give the answer:

76. John had 12 marbles and lost 4 of them.

77. From a flock of 8 birds, a hunter shot 3.

78. Of 16 prisoners, 9 succeeded in escaping.

79. From a bouquet of 13 roses, 8 were taken out.

80. From a flock of 16 sheep, 9 sheep were sold.

81. There were 10 robins and 8 blackbirds in a tree.

82. Don caught his ball 9 times out of 17.

83. Mary paid 12¢ for a book and 5¢ for a pencil.

84. There were 15 wigwams in an Indian village. 9 of them were burned.

85. There were 16 morning glories on a vine. Mary gathered 8 of them.

86. Thomas shook 5 apples from a tree. He gave 4 of them to his sister.

87. Annie had 12¢. She spent 3¢ for an orange.

88. Elizabeth had a 2-cent piece and wanted to buy a ruler which cost 6 cents.

89. Kate was 9 years old when her brother went away from home. When he came back she was 13.

90. An Eskimo had 10 fishhooks. A white man gave him 2 more. He bought 2 more, and then lost 6.

91. Lizzie had 7 cents and found 6 more. She afterwards spent 9 cents for a ribbon.

92. There are 4 stars in the bowl of the Great Dipper and 3 stars in the handle. A cloud covered 2 of them.

93. There were 3 redbirds, 2 yellowbirds, and two bluebirds in an apple-tree.

Counting Backwards

94. Take 17 splints (1 bundle of ten and 7) and take away 2; how many have you? *Ans.* 1 bundle of ten and 5, or 15.

95. Count from 19 backwards by 2's.

Ans. 19, 17, 15, 13, 11, 9, 7, 5, 3, 1.

Count backwards:

96. From 18 by 2's
97. From 19 by 3's
98. From 17 by 3's
99. From 18 by 3's
100. From 17 by 4's
101. From 18 by 4's
102. From 19 by 4's
103. From 16 by 4's
104. From 16 by 5's
105. From 17 by 5's
106. From 18 by 5's
107. From 19 by 5's
108. From 15 by 5's
109. From 19 by 6's

110. Take 24 splints (2 bundles of ten and 4); how can you take away 9 splints?

Ans. Take the band from 1 bundle of ten, leaving 1 bundle of ten; put the 10 single splints and the 4 single splints together, making 14 splints; take away 9 splints; you will have left one bundle of ten and 5 splints, or 15 splints.

How many are:

111. $24 - 8$? $36 - 9$? $55 - 7$? $68 - 9$? $24 - 5$? $36 - 7$? $43 - 8$? $56 - 8$?

112. $55 - 6$? $46 - 8$? $32 - 4$? $27 - 8$? $55 - 7$? $63 - 8$? $25 - 6$? $45 - 7$?

113. $75 - 6$? $85 - 7$? $95 - 8$? $35 - 9$? $46 - 7$? $43 - 9$? $28 - 9$? $22 - 9$?

114. $52 - 3$? $63 - 4$? $74 - 5$? $82 - 6$? $92 - 8$? $71 - 3$? $62 - 5$? $73 - 8$?

Making Change

115. I owe you 3¢; here is a nickel. You may give me the change.

Ans. Say 3; hand me one cent and say 4; another, and say 5.

116. Here is a dime; take 6¢.

Ans. Say 6; hand me a penny and say 7; another, and say 8; and so on. We always begin with the debt and add to it.

117. Here is a quarter; take 17¢.

Ans. Say 17; hand me one cent and say 18; another, 19; another, 20; a nickel, 25. We add to the debt by pennies until we reach the nearest number ending with 5 or 0, and then add by nickels, dimes, or pieces of greater value.

118. Here is a half dollar for that 36¢ hat. Make the change.

119. A boy pays you 49¢ with a quarter and 3 dimes. Make the change.

120. With this dollar I will pay you 78¢; make the change. How much money did you give me?

121. Here are a half dollar and a quarter; I wish to pay you 68¢; make the change.

122. How much money did you just give me?

123. Here is a dollar; I wish to pay you 45¢; make the change. How much money did you give me?

124. Here is a quarter; please give me change so that I can pay Henry 19¢.

Ans. Hand me a dime and say 10; a nickel, 15; a penny, 16; another, 17; another, 18; another, 19; another, 20; a nickel, 25.

125. Here is a dollar; please give me change so that I can pay Mary 59¢. After paying Mary, how much shall I have left?

Addition and Subtraction

Find the number for '?':

126. $8+6-2=?$

127. $3+9-4=?$

128. $7+2-8=?$

129. $8+6-9=?$

130. $19-2-3=?$

131. $17-8-6=?$

132. $18-9-4=?$

133. $16-8-3=?$

Find the number for '?':

134. $8-2+3=?$

135. $9-6+5=?$

136. $7-2+8=?$

137. $5-2+6=?$

138. $9-2-?=4$

139. $8-1-?=2$

140. $6-2-?=1$

141. $7-1-?=4$

Find the number for '?':

142. $36-9-8=?$

143. $45-8-9=?$

144. $52-6-7=?$

145. $31-7-5=?$

146. $75-6-5=?$

147. $84-9-8=?$

148. $92-7-9=?$

149. $30-8-6=?$

Find the number for '?':

150. $36-9+3=?$

151. $48-9+6=?$

152. $73-8+5=?$

153. $24-6+7=?$

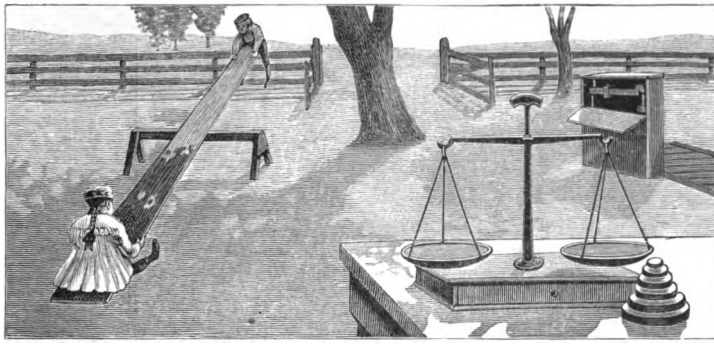
154. $32-8+5=?$

155. $40-7+8=?$

156. $30-5+9=?$

157. $60-8+3=?$

Substances — Weight



158. Balance a pencil or ruler. Does this remind you of any game that you play? How do you play it?

159. Place a pound weight in one tray and a paper bag in the other. Fill the bag with sand until the scales balance. How much does the bag of sand weigh?

160. Remove the bag of sand and place another bag in the tray. Fill the empty bag with dry grass until the scales balance. How much does the bag of grass weigh?

161. Place the bag with one pound of grass in one tray, and the bag with one pound of sand in the other. Do the pound of sand and the pound of grass balance?

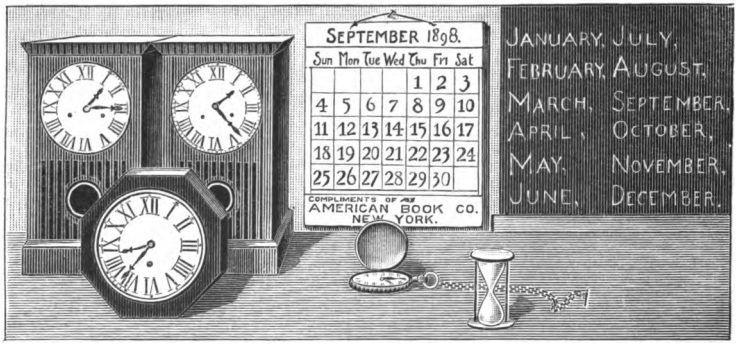
162. How much do you weigh?

163. Place an ounce weight in one tray and weigh out an ounce of sand.

164. Place a pound package in one tray, and ounce packages, one by one, in the other tray, until the scales balance. How many ounces does it take to balance the scales? How many ounces make 1 pound?

165. You may do up a package of sand that weighs a quarter of a pound.

Time



166. How many days in a week? Name them.

167. How many weeks in a month?

168. How many months in a year? Name the months.

169. What day of the week is to-day? was yesterday? will be to-morrow?

170. What month of the year is this month? was last month? will be next month?

171. What day of the month is to-day?

172. How many hours make a day?

173. At what time do you go to bed?

174. At what time do you get up?

175. At what time does school begin in the morning? in the afternoon?

176. How many minutes do you have for recess?

177. How long is the intermission at noon?

178. What time is it at noon? at midnight?

179. How long is it from 9 o'clock until noon? from 1 o'clock until 4 o'clock?

180. How many hours do you spend in school each day?

Time

Draw a clock face with the minute hand at XII and:

181. The hour hand at I. What time is it?
182. The hour hand at III. What time is it?
183. The hour hand at VI. What time is it?
184. The hour hand at VII. What time is it?
185. The hour hand at VIII. What time is it?

Draw the hour hand between XII and I and:

186. The minute hand at I. What time is it?
187. The minute hand at II. What time is it?
188. The minute hand at X. What time is it?
189. The minute hand at III. What time is it?
190. The minute hand at VI. What time is it?
191. The minute hand at IX. What time is it?

192. The minute hand the second space after V. What time is it?

193. What time is it by the first clock in the illustration?

194. What time is it by the second clock in the illustration?

195. What time is it by the third clock in the illustration?

196. Draw a clock face to read 20 minutes to 6 o'clock.

197. How many minute spaces are there on the clock face?

198. How many hour spaces are there on the clock face?

199. While the minute hand goes from XII around to XII again, over how many spaces does the hour hand pass?

200. While the hour hand passes over one minute space, over how many spaces does the minute hand pass?

201. When the hands of a clock are at right angles, how many minute spaces are between them?

MULTIPLICATION

The 45 Combinations

2 3's = 3 2's **$2 \times 3 = 3 \times 2$**

1. Take 6 splints; make piles of 3 splints each; how many piles? how many are 2 3's?

2. Take 6 splints; make piles of 2 splints each; how many piles? how many are 3 2's?

3. Do you see that 2 3's are equal to 3 2's?

4. Show in the picture that 2 sets of 3 cubes are equal to 3 sets of 2 cubes.

5. Show in the picture that the number of girls is the same whether there are 2 groups of 3, or 3 groups of 2.

6. Take 6 splints; give me three of them; give me three of them again; how many times did you give me 3? how many are 2 times 3?

7. Take 6 splints; give me 2 of them; give me 2 again; give me 2 again; how many times did you give me 2? how many are 3 times 2?

8. Do you see that "2 times 3" is equal to "3 times 2"?

The 45 Combinations

$1 \times 1 = 1$	$1 \times 4 = 4$	$1 \times 7 = 7$
$1 \times 2 = 2$	$1 \times 5 = 5$	$1 \times 8 = 8$
$1 \times 3 = 3$	$1 \times 6 = 6$	$1 \times 9 = 9$

9. Read these: 1 times 1 is 1; 1 times 2 is 2; etc.

NOTE. — Observe that 1 times 2 = 2 times 1, 1 times 3 = 3 times 1, etc. Here are 9 combinations; they are already known.

$2 \times 1 = 2$	$2 \times 4 = 8$	$2 \times 7 = 14$
$2 \times 2 = 4$	$2 \times 5 = 10$	$2 \times 8 = 16$
$2 \times 3 = 6$	$2 \times 6 = 12$	$2 \times 9 = 18$

10. Read these: 2 times 1 = 2: 2 times 2 = 4; etc.

11. Read as in addition:

2,	3,	4,	5,	6,	7,	8,	9.

Ans. 4, 6, 8, ··· See p. 31.

12. How many are: 2 2's? 2 3's? 2 4's? 2 5's? 2 6's? 2 7's? 2 8's? 2 9's? 2×9? 2×8? 2×7?

13. Find '?': $2 \times ? = 6$; $? \times 2 = 6$, $2 \times ? = 10$; $2 \times ? = 18$; $? \times 2 = 16$; $2 \times ? = 14$.

14. Mary had 2 paper dolls, and she made 3 dresses for each. How many dresses did she make?

Ans. She made 2 times 3 dresses, or 6 dresses; she made 3 dresses for the first and 3 dresses for the second, or 3 dresses 2 times.

15. A man must buy a pair of shoes for each of two boys. Ask a question and answer it.

16. Tell a story about "$2 \times 5 = 10$."

NOTE. — Since $2 \times 1 = 1 \times 2$, only 8 new combinations are given here; they have been learned in addition.

The 45 Combinations

$3 \times 1 = 3$	$3 \times 4 = 12$	$3 \times 7 = 21$
$3 \times 2 = 6$	$3 \times 5 = 15$	$3 \times 8 = 24$
$3 \times 3 = 9$	$3 \times 6 = 18$	$3 \times 9 = 27$

17. Which of these combinations have you already learned? *Ans.* 3×1, 3×2.

18. Which is the first combination you have not learned?

Ans. 3×3, or 3 3's.

19. How many are 2 3's? Add another 3; how many are 3 3's? 3×3?

20. How many are 2 4's? Add another 4; how many are 3 4's? 3×4?

21. How many are 2 5's? Add another 5; how many are 3 5's? 3×5?

22. In the same way find how many are: 3 6's; 3 7's; 3 8's; 3 9's.

23. How many new combinations here? Memorize them.

24. What is the sum of $6 + 6 + 6$? What is the sum of $3 + 3 + 3 + 3 + 3 + 3$?

25. What does the preceding example illustrate?

Ans. That 3 6's = 6 3's, or that $3 \times 6 = 6 \times 3$.

26. In the same way, show that 3 7's = 7 3's, or that $3 \times 7 = 7 \times 3$.

27. Declare the products rapidly: 3×8 or 8×3; 3×9 or 9×3; 3×4 or 4×3; 3×6 or 6×3; 3×5 or 5×3; 3×7 or 7×3.

28. Find '?': $3 \times ? = 12$; $? \times 5 = 15$; $3 \times ? = 27$; $? \times 8 = 24$; $3 \times ? = 18$; $? \times 7 = 21$.

29. A horse ate 4 quarts of oats at 3 different times during the day. Ask a question and answer it.

30. Tell a story about "$3 \times 6 = 18$."

The 45 Combinations

$4 \times 1 = 4$	$4 \times 4 = 16$	$4 \times 7 = 28$
$4 \times 2 = 8$	$4 \times 5 = 20$	$4 \times 8 = 32$
$4 \times 3 = 12$	$4 \times 6 = 24$	$4 \times 9 = 36$

31. Which of these combinations have you already learned?

32. Which is the first combination you have not learned?

33. How many are 3 4's? Add another 4; how many are 4 4's? 4×4?

34. How many are 3 5's? Add another 5; how many are 4 5's? 4×5?

35. How many are 3 6's? Add another 6; how many are 4 6's? 4×6?

36. How many are 3 7's? Add another 7; how many are 4 7's? 4×7?

37. How many are 3 8's? Add another 8; how many are 4 8's? 4×8?

38. How many are 3 9's? Add another 9; how many are 4 9's? 4×9?

39. How many new combinations here? Memorize them.

40. What is the sum of $7 + 7 + 7 + 7$? What is the sum of $4 + 4 + 4 + 4 + 4 + 4 + 4$?

41. What does the preceding example illustrate?

42. In the same way, show that 4 9's = 9 4's, or that $4 \times 9 = 9 \times 4$.

43. Declare the products rapidly: 4×5 or 5×4; 4×7 or 7×4; 4×6 or 6×4; 4×8 or 8×4; 4×9 or 9×4.

44. Find '?': $4 \times ? = 16$; $4 \times ? = 36$; $? \times 4 = 20$; $? \times 4 = 32$; $4 \times ? = 24$.

45. A man planted 4 rows of tomatoes with 6 in a row. Ask a question and answer it.

The 45 Combinations

$5 \times 1 = 5$	$5 \times 4 = 20$	$5 \times 7 = 35$
$5 \times 2 = 10$	$5 \times 5 = 25$	$5 \times 8 = 40$
$5 \times 3 = 15$	$5 \times 6 = 30$	$5 \times 9 = 45$

46. Which of these combinations have you already learned?

47. Which is the first combination you have not learned?

48. How many are: 4 5's? Add another 5; how many are 5 5's? 5×5?

49. How many are: 5 6's; 5 7's; 5 8's; 5 9's?

50. How many new combinations here? Memorize them.

51. Show that 5 7's = 7 5's, or that $5 \times 7 = 7 \times 5$.

52. Declare the products rapidly: 5×9 or 9×5; 5×7 or 7×5; 5×6 or 6×5; 5×8 or 8×5.

53. Tell a story about "$5 \times 5 = 25$."

$6 \times 1 = 6$	$6 \times 4 = 24$	$6 \times 7 = 42$
$6 \times 2 = 12$	$6 \times 5 = 30$	$6 \times 8 = 48$
$6 \times 3 = 18$	$6 \times 6 = 36$	$6 \times 9 = 54$

54. Which of these combinations have you already learned?

55. Which is the first combination you have not learned?

56. How many are 5 6's? Add another 6; how many are 6 6's? 6×6?

57. Find how many are: 6 7's; 6 8's; 6 9's.

58. How many new combinations here? Memorize them.

59. Show that 6 8's = 8 6's, or that $6 \times 8 = 8 \times 6$.

60. Declare the products rapidly: 6×7 or 7×6; 6×9 or 9×6; 6×8 or 8×6.

NOTE. — The teacher should continue throughout the remaining tables such exercises as: $6 \times ? = 48$; $? \times 9 = 54$.

The 45 Combinations

$7 \times 1 = 7$	$7 \times 4 = 28$	$\mathbf{7 \times 7 = 49}$
$7 \times 2 = 14$	$7 \times 5 = 35$	$\mathbf{7 \times 8 = 56}$
$7 \times 3 = 21$	$7 \times 6 = 42$	$\mathbf{7 \times 9 = 63}$

61. Which of these combinations have you already learned?

62. Which is the first combination you have not learned?

63. How many are 6 7's? Add another 7; how many are 7 7's? 7×7?

64. In the same way, find how many are: 7 8's; 7 9's.

65. How many new combinations here? Memorize them.

66. Show that 7 8's = 8 7's, or that $7 \times 8 = 8 \times 7$.

67. Declare the products rapidly: 7×6; 7×9; 7×5; 3×7; 6×7; 4×7; 7×2; 7×8; 7×7.

68. David sold 8 gallons of milk per day for 7 days. Ask a question and answer it.

$8 \times 1 = 8$	$8 \times 4 = 32$	$8 \times 7 = 56$
$8 \times 2 = 16$	$8 \times 5 = 40$	$\mathbf{8 \times 8 = 64}$
$8 \times 3 = 24$	$8 \times 6 = 48$	$\mathbf{8 \times 9 = 72}$

69. Which of these combinations have you already learned?

70. Which is the first combination you have not learned?

71. How many are 7 8's? Add another 8; how many are 8 8's? 8×8?

72. In the same way, find how many are 9 8's.

73. How many new combinations here? Memorize them.

74. Show that 8 9's = 9 8's, or that $8 \times 9 = 9 \times 8$.

75. Declare the products rapidly: 8×7; 8×8; 8×3; 4×8; 6×8; 8×5; 8×9; 2×8.

The 45 Combinations

$9 \times 1 = 9$	$9 \times 4 = 36$	$9 \times 7 = 63$
$9 \times 2 = 18$	$9 \times 5 = 45$	$9 \times 8 = 72$
$9 \times 3 = 27$	$9 \times 6 = 54$	$9 \times 9 = 81$

76. Which of these combinations have you already learned?

77. Which is the first combination you have not learned?

78. How many are 8 9's? Add another 9; how many are 9 9's? 9×9?

79. How many new combinations here? Memorize it.

80. Declare the products rapidly: 9×8; 6×9; 9×4; 2×9; 9×7; 9×9; 5×9; 9×3.

81. Arrange as below to explain the table of "9 times."

9	9	9	9	9	9	9	9	9	9 9's = 81
9	9	9	9	9	9	9	9	9	8 9's = 72
	18	9	9	9	9	9	9	9	7 9's = 63
		27	9	9	9	9	9	9	6 9's = 54
			36	9	9	9	9	9	5 9's = 45
				45	9	9	9	9	4 9's = 36
					54	9	9	9	3 9's = 27
						63	9	9	2 9's = 18
							72	9	1 9 = 9
								81	

82. In the same way, arrange to explain the table of 8 times.

83. In the same way, arrange to explain the table of 7 times.

84. What will be the cost of 9 loaves of bread at 5¢ each?

85. How much more will 9 oranges cost at 5¢ each than 9 apples at 3¢ each?

The 45 Combinations

Give the answer quickly:

86. Multiply 5, 1, 6, 9, 7, 2, 8, 4, 3; by 2. *Ans.* 10, 2, . . .

87. Multiply 2, 8, 3, 6, 4, 9, 5, 1, 7; by 3. *Ans.* 6, 24, . . .

88. Multiply 3, 9, 4, 7, 5, 1, 6, 2, 8; by 4. *Ans.* 12, 36, . . .

89. Multiply 2, 8, 3, 5, 9, 4, 7, 6, 1; by 5. *Ans.* 10, 40, . . .

90. Multiply 7, 3, 5, 8, 2, 6, 1, 9, 4; by 6. *Ans.* 42, 18, . . .

91. Multiply 6, 2, 1, 4, 3, 7, 9, 8, 5; by 7. *Ans.* 42, 14, . . .

92. Multiply 4, 1, 2, 3, 6, 8, 7, 5, 9; by 8. *Ans.* 32, 8, . . .

93. Multiply 1, 4, 7, 2, 8, 5, 3, 6, 9; by 9. *Ans.* 9, 36, . . .

Give the answer quickly:

94. 2×6, 3×8, 4×5, 9×6, 7×8, 5×9, 4×7, 6×3, 7×9, 4×9, 5×7, 6×9.

95. 3×5, 9×2, 5×5, 2×3, 9×5, 5×2, 4×4, 9×3, 8×4, 5×6, 6×4, 2×2.

96. 6×5, 5×4, 4×7, 3×7, 5×8, 6×6, 3×2, 2×9, 4×3, 9×4, 8×2, 7×7, 2×8, 6×2.

97. 5×3, 6×7, 3×3, 2×5, 7×7, 7×3, 6×8, 7×6, 8×3, 7×2, 2×4, 7×5, 8×9.

98. 3×4, 4×8, 8×5, 3×6, 9×8, 7×4, 3×9, 8×6, 9×9, 2×2, 8×7, 4×6, 2×7, 8×8.

99. 9×9, 8×8, 7×7, 6×6, 5×5, 4×4, 3×3, 2×2, 1×1, 9×7, 8×6, 7×5.

100. 6×4, 5×3, 4×2, 3×1, 2×5, 1×4, 9×5, 8×4, 7×6, 9×5, 9×8, 7×8.

The 45 Combinations

101. How many mittens must grandma knit to give one pair to each of 8 little boys?

102. If balls are worth 9 cents each, how much are 5 balls worth?

103. When eggs cost 6 cents per dozen, how much will 6 dozen cost?

104. How much will 9 pecks of barley cost at 7 cents a peck?

105. Six spoons make a set. How many spoons are there in 9 sets?

106. Fred earns 9 cents a day. How much will he earn in 6 days?

107. If I have 5 apples and cut each into 2 parts, how many pieces shall I have? If I cut each of these pieces into 2 equal parts, how many pieces shall I have?

108. When oranges are worth 3 dollars a box, how much will 9 boxes cost?

109. Carl has 6 marbles, and Fred has 7 times as many. How many marbles has Fred? How many marbles have Carl and Fred together?

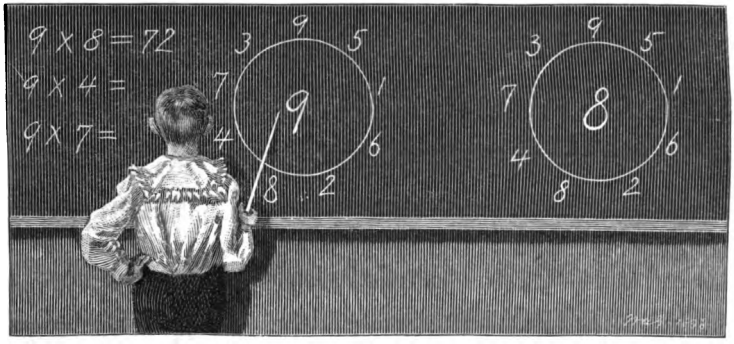
110. Florence has 8 cherries, and Mary has 5 times as many. How many cherries has Mary?

111. Kate has 3 large dolls and 5 times as many small dolls. How many dolls has she altogether?

112. Charles had 7 plants in one basket and 2 in another. He sold them all at the rate of 9 cents apiece. How much did he receive?

113. Nellie buys 6 yards of ribbon at 7 cents a yard and 7 spools of thread at 5 cents a spool. How much more does she pay for the ribbon than for the thread?

The 45 Combinations



114. Point to 9 and 9 and declare the product. In the same way treat: 9 and 5; 9 and 1; and so on around.

115. Place 8 in the center of the circle; declare the products as before.

116. Continue the exercise by placing in the center: 7; 6; 5; 4; 3; 2; 1.

Copy and supply the number for '?':

117.	$7 \times 9 = ?$	$6 \times 7 = ?$	$8 \times 3 = ?$
118.	$9 \times 8 = ?$	$7 \times 5 = ?$	$9 \times 7 = ?$
119.	$6 \times 5 = ?$	$4 \times 5 = ?$	$5 \times 5 = ?$
120.	$8 \times 6 = ?$	$3 \times 7 = ?$	$8 \times 9 = ?$
121.	$8 \times 8 = ?$	$9 \times 3 = ?$	$9 \times 5 = ?$

Copy and supply the number for '?':

122.	$6 \times 9 = ?$	$6 \times 3 = ?$	$5 \times 7 = ?$
123.	$3 \times 4 = ?$	$8 \times 7 = ?$	$7 \times 7 = ?$
124.	$6 \times 6 = ?$	$6 \times 4 = ?$	$4 \times 8 = ?$
125.	$5 \times 8 = ?$	$9 \times 4 = ?$	$4 \times 4 = ?$
126.	$7 \times 4 = ?$	$9 \times 9 = ?$	$3 \times 3 = ?$

Miscellaneous Problems

127. A farmer bought 5 pigs at \$2 each, and sold them at \$9 each. How much was his profit?

Ans. The profit on 1 pig was \$7; on 5 pigs, 5 times \$7, or \$35.

128. How much did the farmer pay for the pigs?

129. For how much did he sell the pigs? What was the gain?

130. If the gain alone is desired, which is simpler, to find the gain on one and multiply by the number, or to find the difference between the entire gain and the entire loss?

131. How much is gained by buying 4 oranges at 2¢ each, and selling them at 5¢ each?

132. How much did the oranges cost? For how much were they sold?

133. A farmer bought 5 pigs at \$2 each, and paid 10¢ each for cartage. What was the entire cost?

134. He sold the pigs at \$9 each, and paid 10¢ apiece to have them delivered. For how much did he sell them? How much did he pay out? How much did he have left?

135. How much is gained by buying 6 oranges for 4¢ each, and selling 3 for 3¢ each, and 3 for 5¢ each?

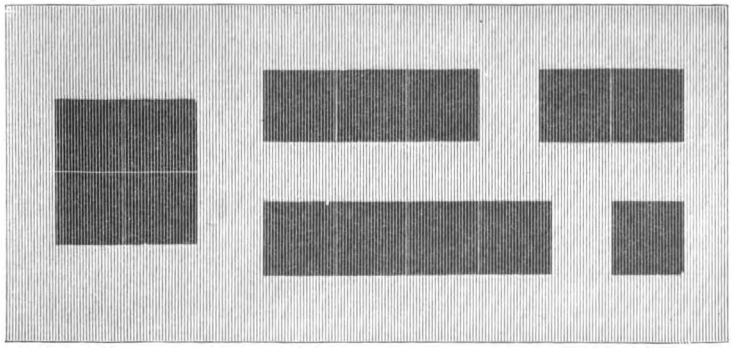
136. How much change must be given when a quarter is offered in payment for 3 oranges at 3¢ each?

137. How much change must be given when a quarter is offered in payment for 2 oranges at 3¢ each, and 6 apples at 2¢ each?

138. John worked 4 hours at 6¢ an hour, and received in payment 2 dimes and a nickel. How much change should he return?

139. Mrs. Smith sold 5 dozen eggs at 9¢ per dozen, and received a half dollar. How much change should she return?

Squares and Rectangles



140. Cut out of paper 12 one-inch squares. The area of each is 1 square inch.

NOTE. — The teacher should explain the meaning of *area*.

141. Place 2 of the squares side by side; they form a rectangle. How much is its area? *Ans.* 2 square inches.

142. Place 3 of the rectangles in a row; they form a rectangle. How much is its area?

143. Form a rectangle by placing 4 of the squares in a row. How much is its area?

144. Place the 4 squares so as to form a large square. How much is its area?

145. In this large square, how many square inches in each row? how many rows?

146. How can you find the area of a rectangle? *Ans.* By multiplying the no. of square inches in each row by the no. of rows.

147. Form a rectangle with 8 squares in a row. How much is its area?

148. Take 8 squares and form a rectangle with 4 squares in a row and 2 rows. How much is its area?

149. Take 12 squares and form a rectangle with 2 rows.

Squares and Rectangles

150. Draw a rectangle 1 foot long and 1 inch wide. What is its area?

Ans. 12 square inches; there are 12 squares in one row.

151. Draw a rectangle 5 inches long and 3 inches wide. What is its area?

Ans. 15 square inches; there are 3 rows and 5 squares in each.

152. If your book is 7 inches long and 6 inches wide, how many square inches on one cover?

153. Form a large square with 12 rows of 12 squares in a row. How much is its area?

Ans. 12 square inches × 12, or 144 square inches.

154. How long is this large square? How wide?

Ans. It is 1 foot long and 1 foot wide. It is 1 foot square, or 1 square foot.

155. How many square inches make 1 square foot?

156. If your desk is 3 feet long and 1 foot wide, how many square feet in its surface?

157. Measure your teacher's desk to the nearest exact foot; measure its width in the same way. How many square feet of cloth would it take to cover the desk?

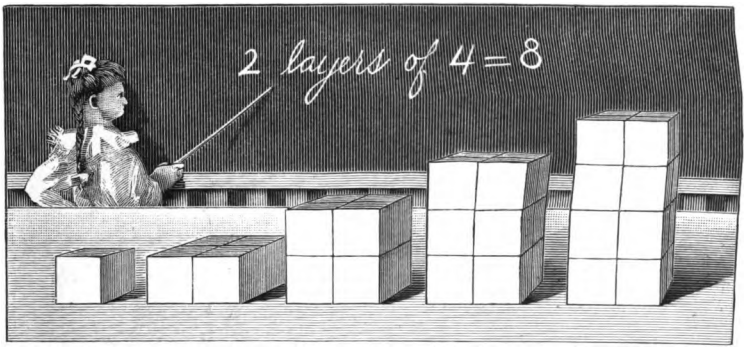
158. Draw on the floor a square 3 feet long and 3 feet wide; divide it into square feet. How many square feet in each row? How many rows?

159. You have just drawn a square yard. How many square feet make a square yard?

160. How many square yards in 45 square feet? in 36 square feet?

161. How many square feet in 8 square yards? in 18 square yards?

Cubes and Prisms



162. Examine a one-inch cube; its length, breadth, and thickness are each 1 inch; its volume is 1 cubic inch.

NOTE. — The teacher should explain the meaning of volume.

163. Place two of the cubes side by side; they form a prism. What is its volume? *Ans.* 2 cubic inches.

164. Place three of the cubes in a row; they form a prism. What is its volume?

165. Form a prism by placing 4 cubes as above; what is its volume?

166. Place another layer of 4 cubes on the first; what is the volume?

167. In this prism, how many cubes in each layer? how many layers? How many are "4 cubic inches × 2"?

168. How can you find the volume of a prism? *Ans.* By multiplying the no. of cubic inches in each layer by the no. of layers.

169. Take 12 cubes, and form a prism with 12 cubes in one layer.

170. Take 12 cubes, and form a prism with 2 layers of 6 each.

DIVISION

How Many in Each Part

$6 \div 2 = 3$ $\qquad$ $6 \div 3 = 2$

1. Take 6 splints, and divide into 2 equal parts; how many in each part? how many are $6 \div 2$?

2. Take 6 splints, and divide into 3 equal parts; how many in each part? how many are $6 \div 3$?

NOTE. — While performing the above exercises, the pupil should have splints in hand.

3. If 6 cubes are divided into 2 equal parts, how many in each part? how many are $6 \div 2$?

4. If 6 cubes are divided into 3 equal parts, how many in each part? how many are $6 \div 3$?

5. If 6 girls are divided into 2 groups of the same number, how many in each group? how many are $6 \div 2$?

6. If 6 girls are divided into 3 groups of the same number, how many in each group? how many are $6 \div 3$?

How Many in Each Part

7. In the picture, 6 cubes are divided in 2 equal parts; what is one part called? *Ans.* One half; written, $\frac{1}{2}$.

8. How many are $\frac{1}{2}$ of 6 cubes? *Ans.* 3 cubes.

9. How many are $\frac{1}{2}$ of 6 girls? how many are $\frac{1}{2}$ of 6?

10. The 6 girls are divided into 3 equal groups; what is one group called? *Ans.* One third; written, $\frac{1}{3}$.

11. How many are $\frac{1}{3}$ of 6 girls? how many are $\frac{1}{3}$ of 6?

12. If a whole is divided into 4 equal parts, what is each part called, and how is it written?

Ans. One fourth, or one quarter; written, $\frac{1}{4}$.

13. What is each part called, and how is it written, when a whole is divided into 5 equal parts? 6 equal parts? 7 equal parts? 8 equal parts? 9 equal parts? See pp. 82 and 83.

$$2 \times 1 = 2$$
$$2 \times 2 = 4$$
. . . . . .

14. Give the table of "2 times" in full.

Ans. $2 \times 1 = 2$, $2 \times 2 = 4$, $2 \times 3 = 6$, etc.

15. Since $2 \times 8 = 16$, how many are $16 \div 2$?

16. How many are: $2 \div 2$? $4 \div 2$? $12 \div 2$? $16 \div 2$? $10 \div 2$? $6 \div 2$? $14 \div 2$? $8 \div 2$?

17. How many are: $\frac{1}{2}$ of 2? $\frac{1}{2}$ of 6? $\frac{1}{2}$ of 12? $\frac{1}{2}$ of 4? $\frac{1}{2}$ of 16? $\frac{1}{2}$ of 8? $\frac{1}{2}$ of 18? $\frac{1}{2}$ of 10? $\frac{1}{2}$ of 14?

18. If you divide 10 apples equally between 2 girls, how many will each receive? *Ans.* $\frac{1}{2}$ of 10 apples, or 5 apples.

19. John can buy 2 oranges for 6 cents. Ask and answer a question.

20. Tell a story about: $\frac{1}{2}$ of 18; $\frac{1}{2}$ of 16; $\frac{1}{2}$ of 8.

How Many in Each Part

$$3 \times 1 = 3$$
$$3 \times 2 = 6$$
$$\cdots\cdots$$

21. Give the table of "3 times" in full.

Ans. $3 \times 1 = 3$, $3 \times 2 = 6$, etc.

22. Since $3 \times 7 = 21$, how many are $21 \div 7$?

23. How many are: $3 \div 3$? $6 \div 3$? $12 \div 3$? $18 \div 3$? $27 \div 3$? $9 \div 3$? $15 \div 3$? $21 \div 3$? $24 \div 3$?

24. How many are: $\frac{1}{3}$ of 6? $\frac{1}{3}$ of 27? $\frac{1}{3}$ of 18? $\frac{1}{3}$ of 12? $\frac{1}{3}$ of 21? $\frac{1}{3}$ of 3? $\frac{1}{3}$ of 9? $\frac{1}{3}$ of 15? $\frac{1}{3}$ of 24?

$$4 \times 1 = 4$$
$$4 \times 2 = 8$$
$$\cdots\cdots$$

25. Give the table of "4 times" in full.

26. How many are: $12 \div 4$? $28 \div 4$? $36 \div 4$? $4 \div 4$? $20 \div 4$? $24 \div 4$? $32 \div 4$? $8 \div 4$? $16 \div 4$?

27. How many are: $\frac{1}{4}$ of 20? $\frac{1}{4}$ of 32? $\frac{1}{4}$ of 12? $\frac{1}{4}$ of 4? $\frac{1}{4}$ of 28? $\frac{1}{4}$ of 36? $\frac{1}{4}$ of 24? $\frac{1}{4}$ of 8? $\frac{1}{4}$ of 16?

$$5 \times 1 = 5$$
$$5 \times 2 = 10$$
$$\cdots\cdots$$

28. Give the table of "5 times" in full.

29. Name the numbers from 5 to 45 that can be divided into 5 equal parts.

30. How many are: $5 \div 5$? $40 \div 5$? $20 \div 5$? $10 \div 5$? $30 \div 5$? $15 \div 5$? $45 \div 5$? $25 \div 5$? $35 \div 5$?

31. How many are: $\frac{1}{5}$ of 15? $\frac{1}{5}$ of 35? $\frac{1}{5}$ of 45? $\frac{1}{5}$ of 30? $\frac{1}{5}$ of 20? $\frac{1}{5}$ of 10? $\frac{1}{5}$ of 5? $\frac{1}{5}$ of 25? $\frac{1}{5}$ of 40?

How Many in Each Part

$6 \times 1 = 6$

. . . . . .

32. Give the table of "6 times" in full.

33. How many are: $12 \div 6$? $30 \div 6$? $54 \div 6$? $18 \div 6$? $36 \div 6$? $24 \div 6$? $48 \div 6$? $6 \div 6$? $42 \div 6$?

34. How many are: $\frac{1}{6}$ of 24? $\frac{1}{6}$ of 12? $\frac{1}{6}$ of 42? $\frac{1}{6}$ of 36? $\frac{1}{6}$ of 54? $\frac{1}{6}$ of 18? $\frac{1}{6}$ of 48? $\frac{1}{6}$ of 36? $\frac{1}{6}$ of 6?

$7 \times 1 = 7$

. . . . . .

35. Give the table of "7 times" in full.

36. How many are: $56 \div 7$? $42 \div 7$? $28 \div 7$? $21 \div 7$? $49 \div 7$? $63 \div 7$? $7 \div 7$? $14 \div 7$? $35 \div 7$?

37. How many are: $\frac{1}{7}$ of 42? $\frac{1}{7}$ of 28? $\frac{1}{7}$ of 21? $\frac{1}{7}$ of 14? $\frac{1}{7}$ of 56? $\frac{1}{7}$ of 63? $\frac{1}{7}$ of 7? $\frac{1}{7}$ of 35? $\frac{1}{7}$ of 49?

$8 \times 1 = 8$

. . . . . .

38. Give the table of "8 times" in full.

39. How many are: $32 \div 8$? $64 \div 8$? $16 \div 8$? $48 \div 8$? $24 \div 8$? $56 \div 8$? $40 \div 8$? $8 \div 8$? $72 \div 8$?

40. How many are: $\frac{1}{8}$ of 8? $\frac{1}{8}$ of 48? $\frac{1}{8}$ of 32? $\frac{1}{8}$ of 64? $\frac{1}{8}$ of 40? $\frac{1}{8}$ of 24? $\frac{1}{8}$ of 16? $\frac{1}{8}$ of 56? $\frac{1}{8}$ of 72?

$9 \times 1 = 9$

. . . . . .

41. How many are: $36 \div 9$? $63 \div 9$? $18 \div 9$? $54 \div 9$? $27 \div 9$? $9 \div 9$? $72 \div 9$? $45 \div 9$? $81 \div 9$?

42. How many are: $\frac{1}{9}$ of 27? $\frac{1}{9}$ of 18? $\frac{1}{9}$ of 63? $\frac{1}{9}$ of 45? $\frac{1}{9}$ of 36? $\frac{1}{9}$ of 72? $\frac{1}{9}$ of 81? $\frac{1}{9}$ of 54? $\frac{1}{9}$ of 9?

How Many in Each Part

How many are:

43. $18 \div 6$? $21 \div 7$? $24 \div 6$? $35 \div 5$? $18 \div 3$? $24 \div 4$? $21 \div 3$?

44. $16 \div 8$? $36 \div 9$? $35 \div 7$? $16 \div 2$? $36 \div 4$? $25 \div 5$? $40 \div 8$?

45. $8 \div 4$? $30 \div 6$? $12 \div 4$? $14 \div 7$? $48 \div 6$? $45 \div 5$? $54 \div 9$?

46. $48 \div 8$? $15 \div 3$? $42 \div 7$? $24 \div 3$? $42 \div 6$? $20 \div 4$? $49 \div 7$?

47. $36 \div 6$? $28 \div 7$? $12 \div 6$? $56 \div 8$? $24 \div 8$? $32 \div 4$? $56 \div 7$?

48. $9 \div 3$? $64 \div 8$? $45 \div 9$? $63 \div 7$? $8 \div 2$? $27 \div 9$? $63 \div 7$?

How many are:

49. $\frac{1}{9}$ of 81? $\frac{1}{6}$ of 30? $\frac{1}{8}$ of 56? $\frac{1}{7}$ of 56? $\frac{1}{9}$ of 54? $\frac{1}{6}$ of 48? $\frac{1}{8}$ of 64? $\frac{1}{3}$ of 9?

50. $\frac{1}{7}$ of 63? $\frac{1}{9}$ of 72? $\frac{1}{8}$ of 40? $\frac{1}{4}$ of 24? $\frac{1}{6}$ of 42? $\frac{1}{8}$ of 32? $\frac{1}{7}$ of 28? $\frac{1}{9}$ of 45?

51. $\frac{1}{9}$ of 36? $\frac{1}{7}$ of 49? $\frac{1}{6}$ of 24? $\frac{1}{9}$ of 63? $\frac{1}{5}$ of 35? $\frac{1}{7}$ of 56? $\frac{1}{5}$ of 45? $\frac{1}{2}$ of 8?

52. $\frac{1}{8}$ of 72? $\frac{1}{7}$ of 42? $\frac{1}{9}$ of 18? $\frac{1}{4}$ of 28? $\frac{1}{9}$ of 45? $\frac{1}{6}$ of 54? $\frac{1}{7}$ of 21? $\frac{1}{9}$ of 27?

53. $\frac{1}{8}$ of 24? $\frac{1}{7}$ of 14? $\frac{1}{5}$ of 40? $\frac{1}{6}$ of 18? $\frac{1}{4}$ of 32? $\frac{1}{2}$ of 10? $\frac{1}{5}$ of 15?

54. $\frac{1}{4}$ of 16? $\frac{1}{3}$ of 21? $\frac{1}{5}$ of 25? $\frac{1}{3}$ of 27? $\frac{1}{4}$ of 36? $\frac{1}{6}$ of 12? $\frac{1}{2}$ of 16? $\frac{1}{5}$ of 30?

How Many in Each Part

State and answer a question:

55. James separated 20 cents into 4 equal parts.

56. A boy had 27 marbles, and put the same number into each of 3 bags.

57. Henry had 45 eggs, and put the same number into each of 5 nests.

58. A farmer separated 48 cattle into 6 equal lots.

59. A woman cut 54 yards of cloth into 9 equal pieces.

60. Mary paid 30 cents for 6 oranges.

61. A liveryman paid $72 for 8 tons of hay.

62. John planted 56 tomato plants in 6 equal rows.

63. Mary picked 8 quarts of currants, and sold them for 56 cents.

64. A vessel sailed 36 miles in 4 hours.

65. Ned paid 36 cents for riding 9 miles.

66. Stephen has 18 doves, and keeps the same number in each of 9 boxes.

67. A boy had 42 cents, and gave $\frac{1}{6}$ of it to a beggar.

68. A man paid $\frac{1}{2}$ of his money for a coat; he had at first $12.

69. Peaches are selling 3 for 6 cents.

70. A man gave $\frac{1}{4}$ of $36 for 3 calves.

71. Jane received $\frac{1}{3}$ of 24 cents for picking 4 quarts of berries.

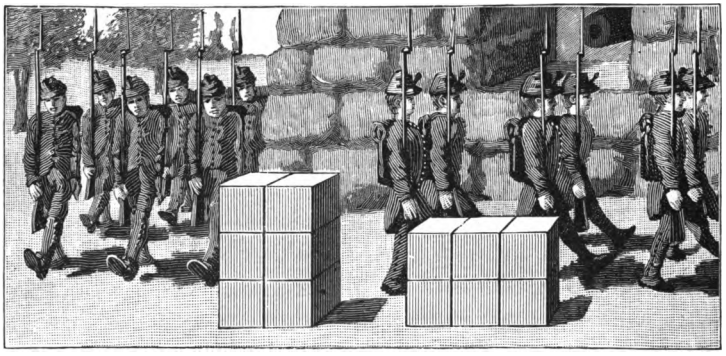
72. A gentleman divided $\frac{1}{5}$ of 45 cents among 3 children.

73. A shoemaker bought for $27 enough leather to make 9 pairs of shoes.

74. A merchant paid $\frac{1}{4}$ of $36 for 3 yards of cloth.

75. A farmer bought for $24 enough lumber to build 2 pens.

How Many Parts — How Many Times



$6 \div 3 = 2$ $6 \div 2 = 3$

76. Take 6 splints; divide into parts of 3 splints each. How many parts? how many 3's in 6? what $= 6 \div 3$?

77. Take 6 splints; take 3 splints away each time until none are left. How many times does 6 contain 3? what $= 6 \div 3$?

78. How many sets, of 2 cubes each, do 6 cubes contain? how many 2's in 6? what $= 6 \div 2$?

79. How many times do 6 cubes contain 2 cubes? what $= 6 \div 2$?

80. How many groups, of 3 soldiers each, do 6 soldiers form? how many 3's in 6?

81. How many groups, of 2 soldiers each, do 6 soldiers form? how many 2's in 6?

82. How many boxes will be required to hold 6 eggs if 2 eggs are placed in each box?

Ans. As many boxes as 2 eggs are contained times in 6 eggs, or 3 boxes. I put 2 eggs in one box, 2 eggs in another, 2 eggs in another, and so on until no eggs are left. By counting, I find there are 3 boxes.

How Many Parts — How Many Times

How many:

83. 2's in 12?
84. 3's in 18?
85. 9's in 63?
86. 5's in 35?
87. 4's in 28?
88. 9's in 36?
89. 6's in 18?

How many times does:

90. 20 contain 4?
91. 42 contain 6?
92. 16 contain 2?
93. 72 contain 9?
94. 64 contain 8?
95. 56 contain 7?
96. 18 contain 9?

How many:

97. 8's in 48?
98. 7's in 14?
99. 4's in 16?
100. 6's in 36?
101. 5's in 30?
102. 7's in 35?
103. 8's in 24?

How many times is:

104. 2 contained in 14?
105. 3 contained in 21?
106. 5 contained in 45?
107. 3 contained in 12?
108. 5 contained in 25?
109. 8 contained in 32?
110. 3 contained in 18?

111. At 2¢ each, how many apples can be bought for 12¢?

Ans. As many apples as 2¢ is contained times in 12¢, or 6 apples.

112. John sets out 56 plants, 7 in a row. Ask and answer the question.

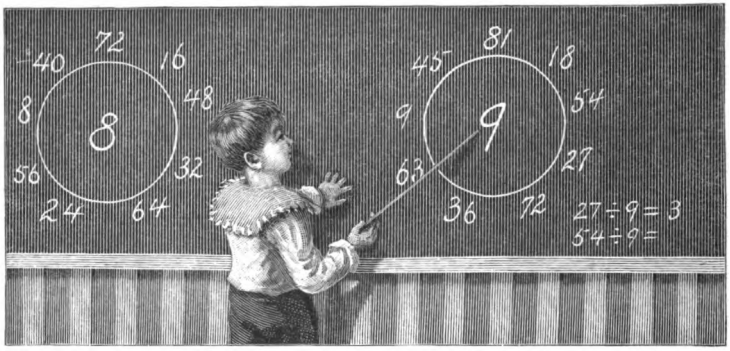
Ans. How many rows did he set out? As many rows as 7 plants is contained times in 56 plants, or 8 rows.

113. A crew can row 6 miles an hour, and have 18 miles in all to go. Ask and answer the question.

114. Make an example about "9 cents is contained in 18 cents."

Ans. If John earns 9 cents in one day, in how many days will he earn 18 cents?

The 45 Combinations



115. Point to 81 and 9 and declare their quotient. In the same way treat: 18 and 9; 54 and 9; and so on around.

116. Place 8 in the center of the circle; count by 8's from 8 to 72 and place the results outside the circle; declare the quotients as before.

117. Continue the exercises as in the last example with: 7; 6; 5; 4; 3; 2; 1.

Copy and supply the number for '?':

118.	$9 \times ? = 45$	$? \times 6 = 54$	$4 \times ? = 28$
119.	$8 \times ? = 72$	$? \times 7 = 63$	$5 \times ? = 35$
120.	$3 \times ? = 12$	$? \times 8 = 56$	$9 \times ? = 81$
121.	$6 \times ? = 42$	$? \times 9 = 45$	$8 \times ? = 64$
122.	$3 \times ? = 24$	$? \times 6 = 30$	$7 \times ? = 49$

Copy and supply the number for '?':

123.	$28 \div ? = 4$	$? \div 4 = 8$	$? \div 5 = 8$
124.	$32 \div ? = 8$	$? \div 7 = 9$	$? \div 7 = 3$
125.	$56 \div ? = 7$	$? \div 9 = 6$	$? \div 6 = 6$
126.	$20 \div ? = 4$	$? \div 8 = 8$	$? \div 5 = 5$
127.	$35 \div ? = 7$	$? \div 4 = 5$	$? \div 6 = 2$

How Many Parts — How Many Times

128. Katie has 2 cents. At 1 cent each, how many sticks of candy can she buy?

129. It takes 3 inches of ribbon to make 1 badge. How many badges will 12 inches make?

130. I see 16 bright eyes. How many children do I see?

131. Harry's mother gives him 2 cents every time he fills her wood box; she has given him 6 cents. How many times has he filled the box?

132. If Frank puts 8 cents into his bank each day, in how many days will there be 64 cents in it?

133. How many cookies can you get for 9 cents, if the price is 3 for a cent?

134. May bought 4 strings of white beads and 2 strings of blue beads; she paid 24 cents for them. How much did she pay for each string?

135. One orange costs 5 cents. How many oranges can you buy for 10 cents?

136. How many pints of peanuts can you buy for 12 cents, when 1 pint costs 6 cents?

137. Jessie had a piece of blue ribbon 14 inches long; she cut it into two-inch pieces. How many pieces were there?

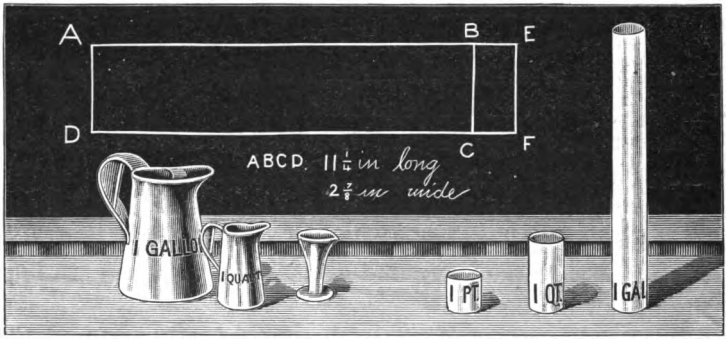
138. If 4 quarts of milk make a pound of cheese, how many pounds of cheese can be made from twelve quarts of milk?

139. I have 12 cents with which to buy 2-cent stamps. How many stamps can I buy?

140. How many pencils can I buy for 18 cents, if 1 pencil costs 3 cents?

141. When coal is 3 dollars a ton, how many tons can you buy for 24 dollars?

Liquids



142. Draw a rectangle 11 inches and 1 quarter long, and 2 inches and 7 eighths wide; draw the flap *BEFC*. Cut out the rectangle *AEFD*, roll *AD* to *BC* and paste *BEFC*. You will have a measure that will hold 1 pint.

143. In the same way, prepare a quart measure. *AB* must be 11 inches and 1 quarter; *AD*, 5 inches and 3 quarters.

144. In the same way, prepare a gallon measure. *AB* must be 11 inches and 1 quarter; *AD*, 23 inches.

145. Compare the pint and quart measures. How many pints make a quart?

146. Compare the quart and gallon measures. How many quarts make a gallon?

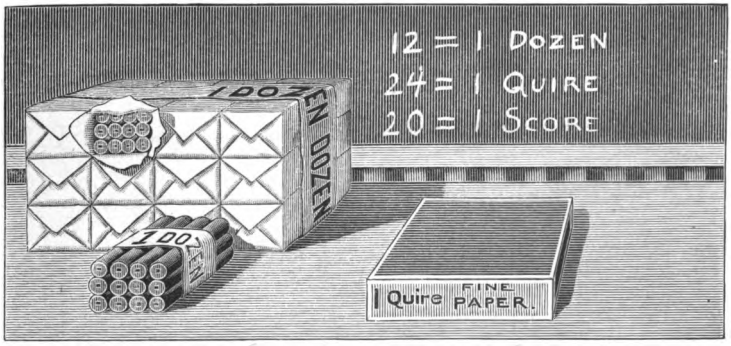
147. How many pints make a gallon?

148. Drop water into a teaspoon and count the drops. How many does it hold? *Ans.* 60.

149. 8 teaspoonfuls of water weigh an ounce. How many doses of 60 drops each in an ounce of medicine?

150. How many doses of 10 drops each in an ounce?

Miscellaneous



151. How do we often buy eggs? How do we buy buttons? *Ans.* By the dozen.

152. How many eggs are half a dozen? a quarter of a dozen?

153. Frank went down town, and bought 1 dozen eggs; on the way home he broke a quarter of them. How many did he break?

154. Mary has 12 girls at her birthday party; how many dozen cookies must she buy that each may have 2 cookies?

155. Henry bought a dozen and a half bananas at 12¢ a dozen; how much did he pay in all?

156. Jane bought a quire of paper; how many sheets did she buy? *Ans.* 24.

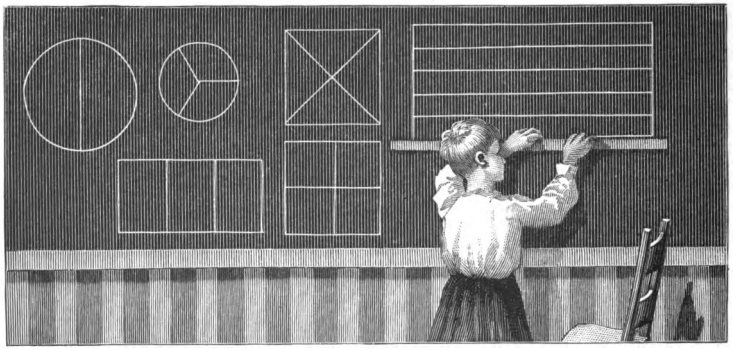
157. How many envelopes will be needed for half a quire of paper?

158. How much will a fourth of a quire of paper cost at 12¢ for 24 sheets?

159. John saw a score of people; how many people did he see? *Ans.* 20 people.

FRACTIONS

A Whole into Equal Parts



1. Draw a circle and divide by a straight line into two equal parts. The straight line passes through the center and is called the diameter.

2. Draw a circle and divide by straight lines from the center into three equal parts; each line is a radius.

3. Draw a square and divide into two equal parts by a line drawn from one corner to the opposite corner; the line is a diagonal.

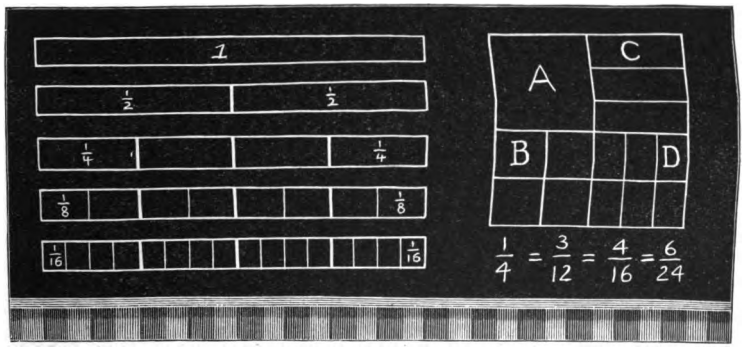
4. Draw a rectangle and divide it into three equal parts.

5. Draw a square and divide it into four equal parts by diagonals.

6. Draw a square and divide it by horizontal (side to side) and vertical (up and down) lines into four equal parts.

7. Draw a rectangle and divide it by horizontal lines into five equal parts.

Relations — Each Part to the Whole



8. Draw a rectangle and divide into three equal parts. What part of the whole is each portion? Write $\frac{1}{3}$ on each part. How many thirds make a whole?

9. Draw a rectangle and divide into five equal parts. What is each part called? How many fifths make a whole?

10. On each fifth of the rectangle place two dots. How many dots in all? How many dots on a fifth? What is $\frac{1}{5}$ of 10 dots?

11. Draw a rectangle and divide into eight equal parts. Each part is what part of the whole? Write $\frac{1}{8}$ on each part.

12. Take 16 beans; put one on the first part of this rectangle; another on the second; and so on until all the beans are used. How many beans on each part? What is $\frac{1}{8}$ of 16?

13. In the same way as in Ex. 12, show that $\frac{1}{5}$ of 15 is 3.

14. Draw a square; divide and letter as above.

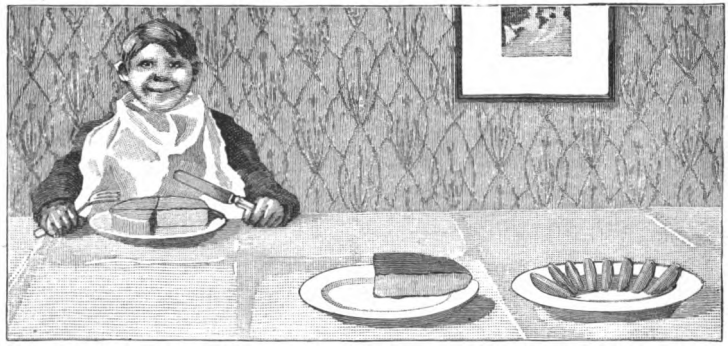
15. What is the relation of A to the whole?

Ans. A is $\frac{1}{4}$ of the whole.

16. What is the relation of B to the whole? of C? of D?

17. If the whole is 24, what is A? B? C? D?

Relation — Several Equal Parts to the Whole



18. If a cake is divided into four equal parts, and Henry takes three parts, how many parts will be left?

19. Answer this in another way.

Ans. If Henry takes $\frac{3}{4}$ of a cake, $\frac{1}{4}$ will be left.

20. If an orange is divided into 8 equal parts, what part of the whole is each portion?

21. James has 5 pieces; what part of the whole has he?

Ans. $\frac{5}{8}$.

22. How many pieces are left? What part of the whole?

23. A man gives 6¢ to James, 2¢ to Henry, and 3¢ to Philip. How much does he give to all?

24. What part of the whole does he give to James? to Henry? to Philip?

25. Draw a circle and divide into 8 equal parts by lines from the center; erase 3 of the parts. What part of the whole did you erase? What part of the whole is left?

26. Draw a square and divide into four equal parts by horizontal and vertical lines; in the same way divide each of the small squares into four equal parts; erase five of the smallest squares. What part of the original whole is left?

Problems

27. If there were 24 children in your class and $\frac{1}{3}$ of the class were promoted, how many children were promoted? What part of the class failed? How many children failed?

28. Frank had 12 cents in his pocket. He spent $\frac{3}{4}$ of his money. How many cents remained?

29. There are 12 eggs in a dozen. I use $\frac{1}{4}$ of the dozen for my cake. How many eggs do I use?

30. John has 12 marbles. He gives Ned $\frac{1}{4}$ of them, and Fred $\frac{1}{4}$ of them. How many does he give to each? How many does he give to both? How many has he left?

31. When butter is 12¢ a pound, how much will 1 and $\frac{1}{2}$ pounds cost?

32. John had 9¢; his father gave him 6¢ more; and he spent $\frac{1}{3}$ of his money for candy. How much did he spend?

33. During the last 6 days, $\frac{1}{3}$ of the days have been fair, $\frac{1}{2}$ cloudy, and $\frac{1}{6}$ stormy. How many fair days? How many cloudy days? How many stormy days?

34. Ross had 8 sticks of candy. He gave 4 to Tom and ate half a stick. How many had he left?

35. Three, three, and a third of three are how many?

36. How much will one half pound of candy cost, at fourteen cents a pound?

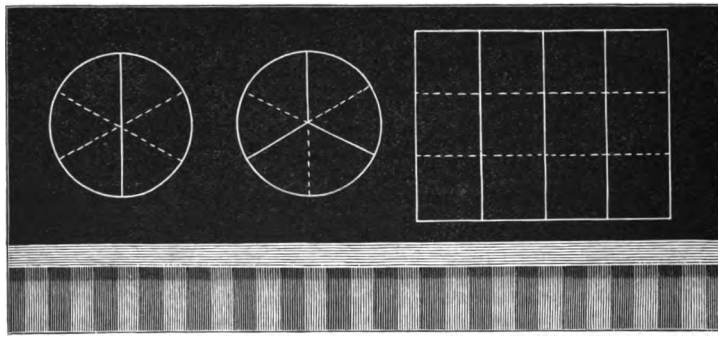
37. There are 9 eggs in one basket and $\frac{2}{3}$ as many in another. How many in both?

38. In a coop were 20 chickens. After $\frac{3}{4}$ of them were sold, how many remained?

39. Kate had a pear and gave her sister $\frac{2}{3}$ of it. What part of the pear had she left for herself?

40. Would you rather have $\frac{2}{3}$ of 6 apples, or $\frac{1}{2}$ of 8 apples?

Difference of Parts



41. Draw on paper two equal circles; by radii (lines from the center) divide one into 2 equal parts and the other into 3 equal parts. Which is the larger, $\frac{1}{2}$ or $\frac{1}{3}$?

42. By dotted radii divide each part of the first circle into 3 equal parts and each part of the second into 2 equal parts. What is each part of the first called? of the second?

43. How many $\frac{1}{6}$'s make $\frac{1}{2}$? How many $\frac{1}{6}$'s make $\frac{1}{3}$?

44. How much larger is $\frac{1}{2}$ than $\frac{1}{3}$?

Ans. $\frac{1}{6}$; $\frac{1}{2}$ equals $\frac{3}{6}$ and $\frac{1}{3}$ equals $\frac{2}{6}$; the difference is $\frac{1}{6}$.

45. In the same way as in Ex. 41, determine which is the larger, $\frac{1}{3}$ or $\frac{1}{4}$.

46. By dotted radii divide each $\frac{1}{3}$ into 4 equal parts and each $\frac{1}{4}$ into 3 equal parts. How much larger is $\frac{1}{3}$ than $\frac{1}{4}$?

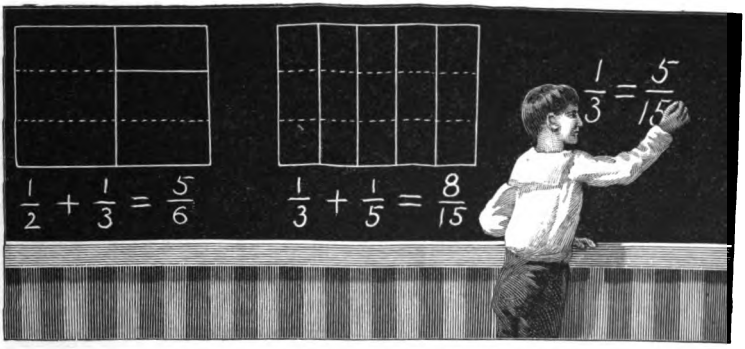
47. Divide a rectangle into 3 equal parts by dotted horizontal lines; into 4 equal parts by continuous vertical lines. How large is each part?

48. How many of the parts does $\frac{2}{3}$ contain? *Ans.* 8 parts, $\frac{8}{12}$.

49. How many of the parts does $\frac{3}{4}$ contain? *Ans.* 9 parts, $\frac{9}{12}$.

50. How much larger is $\frac{3}{4}$ than $\frac{2}{3}$? *Ans.* $\frac{1}{4}$ is $\frac{1}{12}$ larger.

Sum of Parts



51. How much is $\frac{1}{2}$ and $\frac{1}{3}$? Draw a rectangle and divide as above; $\frac{1}{2} = \frac{3}{6}$; $\frac{1}{3} = \frac{2}{6}$; their sum is $\frac{5}{6}$.

52. How much is $\frac{2}{3}$ and $\frac{2}{5}$? Draw a rectangle and divide as above.

53. George had $\frac{1}{6}$ of a pie, and Henry $\frac{1}{5}$; which had the more? How much more?

54. Mary received $\frac{1}{3}$ of 12¢, and Jane $\frac{1}{4}$ of 12¢. Which received the more? How much more? What part of 12¢ more?

55. John earned $\frac{1}{2}$ of a dollar, and James $\frac{1}{4}$ of a dollar. How much did they both earn?

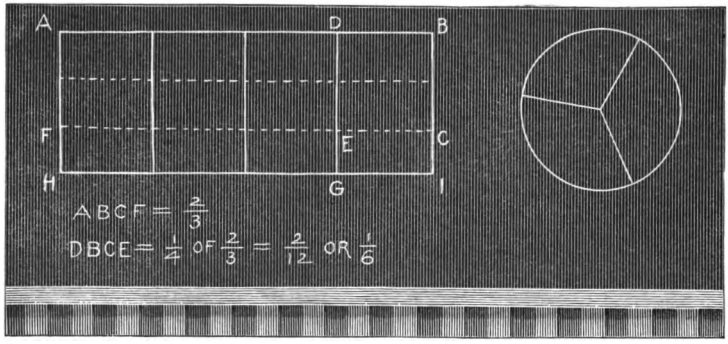
56. If Janet has 1 apple and gives $\frac{5}{8}$ of it to Matilda, and $\frac{1}{4}$ to Martha, how much has she then?

57. A man had $32. He spent $\frac{7}{8}$ of it for a horse. How much money had he remaining? What part of the whole?

58. Draw a line $\frac{3}{4}$ inch + $\frac{7}{8}$ inch long. How many eighths long is the line? What is the sum of $\frac{7}{8}$ and $\frac{3}{4}$?

59. Find the sum of $\frac{7}{8}$ and $\frac{3}{4}$ by drawing and dividing a rectangle.

A Part of a Part



60. What is $\frac{1}{4}$ of $\frac{2}{3}$?

Ans. I draw a rectangle, divide it into 3 equal parts, and letter $\frac{2}{3}$ of it *ABCF*. I divide the $\frac{2}{3}$ into 4 equal parts, and letter one part *DBCE*. *DBCE* = $\frac{2}{12}$, or $\frac{1}{6}$ of the whole rectangle. Therefore $\frac{1}{4}$ of $\frac{2}{3}$ = $\frac{1}{6}$.

61. What is $\frac{3}{4}$ of $\frac{2}{3}$?

Ans. $\frac{3}{4}$ of $\frac{2}{3}$ is *ADEF*; *ADEF* = $\frac{6}{12}$, or $\frac{1}{2}$ of the whole. Therefore $\frac{3}{4}$ of $\frac{2}{3}$ = $\frac{1}{2}$.

62. In a similar way, find $\frac{3}{4}$ of $\frac{3}{5}$.

63. Find $\frac{3}{4}$ of $\frac{2}{3}$ in another way.

Ans. 1 foot = 12 inches; $\frac{2}{3}$ of 12 inches = 8 inches; $\frac{3}{4}$ of 8 inches = 6 inches; 6 inches = $\frac{1}{2}$ of 12 inches. Therefore $\frac{3}{4}$ of $\frac{2}{3}$ = $\frac{1}{2}$.

64. Find $\frac{3}{4}$ of $\frac{3}{5}$ in another way.

Ans. There are 20 school days in a month; $\frac{3}{5}$ of 20 days = 12 days; $\frac{3}{4}$ of 12 days = 9 days; 9 days = $\frac{9}{20}$ of 20 days. Therefore $\frac{3}{4}$ of $\frac{3}{5}$ = $\frac{9}{20}$.

65. Find the value of $\frac{3}{4}$ of $\frac{1}{2}$.

66. What is: $\frac{1}{2}$ of $\frac{5}{6}$? $\frac{1}{4}$ of $\frac{3}{8}$? $\frac{2}{3}$ of $\frac{5}{6}$?

67. What is: $\frac{2}{3}$ of $\frac{5}{8}$? $\frac{1}{2}$ of $\frac{2}{3}$? $\frac{1}{3}$ of $\frac{3}{4}$?

A Part Divided by a Part

68. What is $\frac{2}{3} \div \frac{3}{4}$?

Ans. I draw a rectangle, divide by dotted lines into 3 equal parts, and letter $\frac{2}{3}$ of it $ABCF$. I divide the rectangle into 4 equal parts and letter $\frac{3}{4}$ of it $ADGH$. $ABCF = 8$ 12ths; $ADGH = 9$ 12ths; 8 12ths $\div$ 9 12ths $= \frac{8}{9}$. Therefore $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

69. In a similar way, find the value of $\frac{3}{4} \div \frac{3}{5}$.

70. Find the value of $\frac{2}{3} \div \frac{3}{4}$ in another way.

Ans. 1 foot $= 12$ inches; $\frac{2}{3}$ of 12 inches $= 8$ inches. $\frac{3}{4}$ of 12 inches $= 9$ inches; 8 inches $\div$ 9 inches $= \frac{8}{9}$. Therefore $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

71. Find the value of $\frac{3}{4} \div \frac{3}{5}$ in another way.

Ans. There are 20 school days in a month; $\frac{3}{4}$ of 20 days $= 15$ days; $\frac{3}{5}$ of 20 days $= 12$ days; 15 days $\div$ 12 days $= \frac{15}{12}$, or $\frac{5}{4}$. Therefore $\frac{3}{4} \div \frac{3}{5} = \frac{5}{4}$.

72. What is $\frac{1}{2} \div \frac{5}{6}$? $\frac{1}{4} \div \frac{3}{8}$? $\frac{2}{3} \div \frac{5}{6}$? $\frac{3}{4} \div \frac{7}{8}$?

73. If $\frac{1}{3}$ of an article costs 6¢, how much will the whole cost?

Ans. I draw a circle, divide it into 3 equal parts, and write 6¢ on each part. If $\frac{1}{3}$ of an article costs 6¢, the whole will cost 3 times 6¢, or 18¢.

74. If $\frac{2}{3}$ of an article costs 6¢, how much will the whole cost?

Ans. I draw a circle and divide it into 3 equal parts. Since 2 of these parts cost 6¢, 1 part will cost $\frac{1}{2}$ of 6¢, or 3¢. I write 3¢ on each part; the whole will cost 3 times 3¢, or 9¢.

Or, if $\frac{2}{3}$ of an article costs 6¢, $\frac{1}{3}$ of the article will cost $\frac{1}{2}$ of 6¢, or 3¢; the whole will cost 3 times 3¢, or 9¢.

75. If $\frac{3}{4}$ of a melon costs 12¢, how much will the whole cost?

STUDY OF RELATIONS

STATEMENT. — A boy had 9 cents; after losing 5 cents he had 4 cents left.

1. Omit "9 cents" and state the example.

Ans. After losing 5 cents a boy had 4 cents left. How much had he at first?

2. Omit "5 cents" and state the example.

Ans. A boy had 9 cents; after losing a certain sum he had 4 cents left. How much money did he lose?

3. Omit "4 cents" and state the example.

Ans. A boy had 9 cents; he lost 5 cents. How much had he left?

Example I. — After losing 5 cents a boy had 4 cents left. How much had he at first?

4. What is known? *Ans.* He lost 5 cents; he had 4 cents left.

5. What is required? *Ans.* How many cents he had at first.

6. How can you find how much he had at first?

Ans. By adding what he lost and what he had left.

7. Analyze.

Ans. He had left the sum of 5 cents and 4 cents, or 9 cents.

Example II. — A boy had 9 cents; after losing a certain sum he had 4 cents left. How much money did he lose?

8. What is known? What is required?

9. How can you find how much he lost?

10. Analyze. *Ans.* He lost 9 cents minus 4 cents, or 5 cents.

Study of Relations

STATEMENT. — If 1 apple costs 3 cents, 4 apples will cost 12 cents.

11. Omit 12 cents and state the example ; number it III.

12. Omit 4 apples and state the example; number it IV.

13. Omit 3 cents and state the example ; number it V.

Example III. — If 1 apple costs 3 cents, how much will 4 apples cost?

14. What is known? What is required?

15. How can you find how much all cost?

16. Analyze. *Ans.* Since 1 apple costs 3 cents, 4 apples will cost 4 times 3 cents, or 12 cents.

Example IV. — If 1 apple costs 3 cents, how many apples can be bought for 12 cents?

17. What is known? What is required?

18. How can you find the number of apples?

19. Analyze. *Ans.* Since 1 apple costs 3 cents, as many apples can be bought for 12 cents as 3 cents is contained times in 12 cents, or 4 apples.

Example V. — If 4 apples cost 12 cents, how much will 1 apple cost?

20. What is known? What is required?

21. How can you find the cost of 1 apple?

22. Analyze. *Ans.* Since 4 apples cost 12 cents, 1 apple will cost $\frac{1}{4}$ of 12 cents, or 3 cents.

STATEMENT. — At 3 for 5¢, 6 bananas can be bought for 10¢.

23. Omit 10¢ and state the problem ; number it VI.

24. Omit 6 bananas and state the problem; number it VII.

25. Omit 5¢ and state the problem ; number it VIII.

26. Omit 3 and state the problem ; number it IX.

Study of Relations

Example VI. — At 3 for 5¢, how much will 6 bananas cost?

27. What is known? What is required?

28. What is the relation?

Ans. 6 bananas will cost twice as much as 3 bananas.

29. Analyze. *Ans.* Since 6 bananas cost twice as much as 3 bananas, 6 bananas will cost 2 times 5¢, or 10¢.

30. State two relations. *Ans.* 1 banana will cost $\frac{1}{3}$ of 5¢; 6 bananas will cost 6 times as much as 1 banana.

31. Analyze from these relations. *Ans.* Since 3 bananas cost 5¢, 1 banana will cost $\frac{1}{3}$ of 5¢, or $\frac{5}{3}$¢; 6 bananas will cost 6 times $\frac{5}{3}$¢, or 10¢.

32. Which form of analysis is the better for this problem?

Ans. The former.

Example VII. — At 3 for 5¢, how many bananas can be bought for 10¢?

33. What is known? What is required? What is the relation?

Ans. The relation is, that 10¢ will buy 2 times as many bananas as 5¢.

34. Analyze, using this relation.

35. Analyze, using two relations: that 1 banana will cost $\frac{1}{3}$ of 5¢; that as many bananas can be bought for 10¢ as $\frac{5}{3}$¢ is contained times in 10¢. Which do you prefer?

Example VIII. — How much will 3 bananas cost if 6 bananas cost 10¢?

36. See Ex. VI. Analyze by one relation; by two relations. Which do you prefer?

Example IX. — How many bananas can be bought for 5¢ if 6 bananas cost 10¢?

37. See Ex. VII. — Analyze by one relation; by two relations. Which do you prefer?

Study of Relations

Example X. — If I buy 6 oranges at 3¢ each and sell them at 5¢ each, how much do I make?

38. Analyze by these relations: the gain on 1 orange is 2¢; the gain on 6 oranges is 6 times 2¢.

39. Analyze by these relations: the cost of 6 oranges is 18¢; the selling price of 6 oranges is 30¢; the gain is the difference. Which do you prefer?

40. James had 7¢ and spent 2¢. How much had he left?

41. After spending 8¢, John had 5¢ left. How much had he at first?

42. Henry had 12¢; after spending a certain number of cents, he had 9¢ left. How much did he spend?

43. If 1 orange costs 5¢, how much will 6 oranges cost?

44. At 2¢ each, how many apples can be bought for 10¢?

45. If 3 apples cost 12¢, how much will 1 apple cost?

46. At 2 for 5¢, how many pears can be bought for 20¢?

47. At 2 for 5¢, how much will 6 pears cost?

48. How much is gained by buying 12 calves at $5 each, and selling them at $7 each?

49. Mary exchanged 3 chickens at 20¢ each for 5 yards of cloth. How much did she pay for 1 yard?

50. Jane exchanged 2 chickens at 20¢ each for cloth at 8¢ a yard. How many yards did she buy?

51. How many chickens at 20¢ each must be given in exchange for 6 yards of cloth at 10¢ a yard?

52. If 5 chickens are given in exchange for 10 yards of cloth at 10¢ a yard, how much is one chicken worth?

53. Mary sent 6 dozen eggs to market, and sold them at 12¢ a dozen. If she paid 1¢ a dozen for cartage, how much did she have left after paying cartage?

NUMBERS BY LETTERS

Problems

1. A boy had 2 marbles and bought 3 marbles. How many had he then? *Ans.* $(2 + 3)$ marbles.

2. A boy had a marbles and bought b marbles. How many had he then? *Ans.* $(a + b)$ marbles.

3. A girl had 9 dishes and broke 2 of them. How many were left? *Ans.* $(9 - 2)$ dishes.

4. A girl had c dishes and broke d of them. How many were left? *Ans.* $(c - d)$ dishes.

5. At $3\,¢$ each, how much will 2 apples cost?

Ans. 2 times $3\,¢$, or $6\,¢$.

6. At $3\,¢$ each, how much will x apples cost?

Ans. x times $3\,¢$, or $3x\,¢$.

7. If 2 apples cost $6\,¢$, how much will 1 apple cost?

Ans. $\frac{1}{2}$ of $6\,¢$, or $3\,¢$.

8. If x apples cost $6\,¢$, how much will 1 apple cost?

Ans. $\frac{1}{x}$ of $6\,¢$, or $\frac{6}{x}¢$ (6 divided by x cents).

9. At $3\,¢$ each, how many apples can be bought for $6\,¢$?

Ans. As many apples as $3\,¢$ is contained times in $6\,¢$, or 2 apples.

10. At x cents each, how many apples can be bought for $6\,¢$?

Ans. As many apples as $x\,¢$ is contained times in $6\,¢$, or $\frac{6}{x}$ apples.

Problems

11. Take some number; multiply by 2; multiply by 6; divide by 3; the result is 20. What is the number?

Let x = *the number*
$2x$ = *the number multiplied by 2*
$12x$ = *the result multiplied by 6*
$4x$ = *the result divided by 3*
20 = *the result divided by 3*
$\therefore 4x = 20$
$x = 5$, *the number*

12. Prove that 5 is the number.

Ans. $5 \times 2 = 10$; $10 \times 6 = 60$; $60 \div 3 = 20$.

13. A man bought 12 animals, cows and horses; the number of cows was twice the number of horses. How many of each did he buy? Prove the answers.

Let x = *the number of horses*
$2x$ = *the number of cows*
$3x$ = *the number in all*
12 = *the number in all*
$\therefore 3x = 12$
$x = 4$, *the number of horses*
$2x = 8$, *the number of cows*

PROOF. — 1st, there must be 12 animals; $4 + 8 = 12$. 2nd, the no. of cows must be twice the no. of horses; 8 is twice 4.

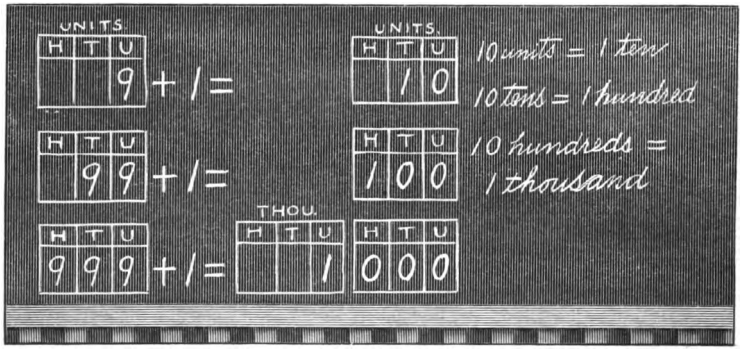
14. Take some number; multiply by 6; divide by 3; multiply by 5; divide by 2; the result is 15. What is the number? Prove your answer.

15. John had a certain number of cents; Henry had 3 times as much as John; both had 20¢. How much had each? Prove the answers.

16. A man paid a certain sum for a knife, 3 times as much for a book, and 4 times as much for a chair; for all he paid $24. How much did he pay for each? Prove the answers.

PART SECOND

NOTATION AND NUMERATION



Numbers Less than Twenty

Numbers less than twenty are named and written as follows:

Naught,	0	Ten,	10
One,	1	Eleven,	11
Two,	2	Twelve,	12
Three,	3	Thirteen,	13
Four,	4	Fourteen,	14
Five,	5	Fifteen,	15
Six,	6	Sixteen,	16
Seven,	7	Seventeen,	17
Eight,	8	Eighteen,	18
Nine,	9	Nineteen,	19

1. For what does ten stand? *Ans.* 1 ten 0.
2. For what does eleven stand? *Ans.* 1 ten 1.
3. For what does twelve stand? Thirteen? Fourteen?

Numbers Twenty to One Hundred

Numbers less than one hundred are named and written as follows:

Twenty,	20	–,	–,	–,	–,	–,	–,	Ninety,	90
Twenty-one,	21	–,	–,	–,	–,	–,	–,	Ninety-one,	91
Twenty-two,	22	–,	–,	–,	–,	–,	–,	Ninety-two,	92
Twenty-three,	23	–,	–,	–,	–,	–,	–,	Ninety-three,	93
Twenty-four,	24	–,	–,	–,	–,	–,	–,	Ninety-four,	94
Twenty-five,	25	–,	–,	–,	–,	–,	–,	Ninety-five,	95
Twenty-six,	26	–,	–,	–,	–,	–,	–,	Ninety-six,	96
Twenty-seven,	27	–,	–,	–,	–,	–,	–,	Ninety-seven,	97
Twenty-eight,	28	–,	–,	–,	–,	–,	–,	Ninety-eight,	98
Twenty-nine,	29	–,	–,	–,	–,	–,	–,	Ninety-nine,	99

4. For what does the first column of dashes stand?

Ans. Thirty, thirty-one, thirty-two, and so on.

5. For what does the second column stand? the third? the fourth? the fifth? the sixth?

6. For what does twenty stand? *Ans.* 2 ten 0.

7. For what does twenty-one stand? twenty-five? thirty? thirty-eight? sixty-nine?

8. For what does thirty-six stand?

9. In writing, how is "3 ten 6" abbreviated?

Ans. 36; the word "ten" is omitted, and 3, in this position, means "3 tens."

10. In 33, does the first 3, counting from the right, stand for the same as the second 3?

Ans. No! Counting from the right, the first 3 represents 3 units, and is in units' *order;* the second 3, 3 tens, and is in tens' *order.*

11. Read: 75; 69; 23; 89; 70; 42; 50; 87.

12. Write: sixty-two; eighty; eighty-two; ninety; seventy-nine.

Numbers One Hundred to One Thousand

Numbers between one hundred and one thousand are named and written as follows :

One hundred,	100
One hundred one,	101
. . . .	. . .
One hundred ninety-nine,	199
Two hundred,	200
. . . .	. . .
Nine hundred,	900
Nine hundred one,	901
. . . .	. . .
Nine hundred ninety-nine,	999

13. Supply some of the numbers represented by the first line of dots.

Ans. One hundred two (102), one hundred three (103).

14. By the second line of dots ; by the third line.

15. For what does "one hundred" stand ?

Ans. 1 hundred 1 ten 0.

16. For what does "one hundred ninety-nine" stand ? two hundred ? four hundred fifty-six ? seven hundred eighty-two ?

17. In writing, how is "7 hundred 8 ten 2" abbreviated ?

Ans. 782; 7, in this position, means 7 hundreds.

18. In 333, does the first 3, counting from the right, stand for the same as the second 3 and the third 3 ?

Ans. No! Counting from the right, the first 3 represents 3 units and is in units' *order;* the second 3, 3 tens and is in tens' *order;* the third 3, 3 hundreds and is in hundreds' *order.*

19. Read : 360; 705; 815; 920; 616; 325; 408.

20. Write : five hundred six ; six hundred seventy-eight ; nine hundred fifteen ; two hundred six.

Numbers Beginning with One Thousand

Numbers beginning with one thousand are named and written as follows:

1 thousand,	1,000
1 thousand 1,	1,001
. . . .	. . . .
999 thousand,	999,000
999 thousand 1,	999,001
. . . .	. . . .
999 thousand 999,	999,999
1 million,	1,000,000
1 million 1,	1,000,001
. . . .	. . . .
999 million 999 thousand 999,	999,999,999
1 billion,	1,000,000,000
. . . .	. . . .

21. Supply some of the numbers represented by the first line of dots. *Ans.* 1 thousand 2 (1002), 1 thousand 3 (1003).

22. Supply some of the numbers represented by the second line of dots; by the third; by the fourth.

23. For what does "one hundred sixty-two thousand, two hundred" stand? *Ans.* 162 thousand 200.

24. For what does "three hundred million, three hundred thousand, three hundred" stand?

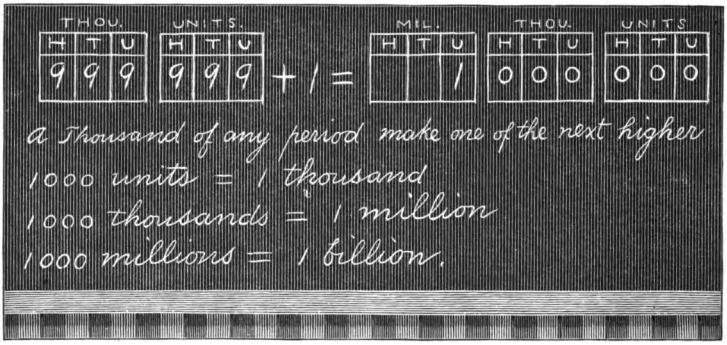
Ans. 300 million 300 thousand 300.

25. In writing, how is "300 million 300 thousand 300" abbreviated? *Ans.* 300,300,300.

26. In 300,300,300 does the first 300, counting from the right, stand for the same as the second 300 and the third 300?

Ans. No! Counting from the right, the first 300 represents 300 units and is in units' *period;* the second, 300 thousands and is in thousands' *period;* the third, 300 millions and is in millions' *period.*

Numeration — More than One Period.



I. *A number is pointed off from right to left into periods of three figures each.*

Copy and point off into periods of three figures each:

a.	3245	*d.*	900025
b.	26070	*e.*	3087650
c.	396281	*f.*	11870368

27. How many figures may there be in the left-hand period? How many must there be in each of the other periods?

28. Analyze: 11,870,368; 3,087,650; 3245.

Ans. 11 is in the third period from the right; 870, in the second period; 368, in the first period.

II. *The periods are named from right to left to learn the name of the left-hand period. They are: units; thousands; millions; . . .*

29. Beginning at the right, point to and name the periods; in 3,087,650; in 27,070; in 11,870,368.

Ans. Pointing to 650, say units; pointing to 087, say thousands; pointing to 3, say millions.

Numeration — More than One Period

III. *The periods are read and named, but the period,* 000, *is not read, and units' period is not named.*

30. Read the first period in 3,087,650. *Ans.* 3.

31. Name the first period. *Ans.* Millions.

32. How was the name of this period found?

Ans. By pointing to and naming the periods from right to left.

33. Read and name the second period in the above.

Ans. 87 thousand.

34. Read, but do not name the third period of the above.

Ans. 650.

35. Read and name the periods: in 3,245; in 26,070.

36. Read and name the periods in 100,396,281.

37. Read 11000368.

Ans. 11 million, 368. Pointing off into periods of three figures each, 11,000,368; numerating to learn the name of the left-hand period, units, thousands, millions; reading and naming the periods, 11 million, 368.

NOTE. — The plural form of the period's name should be used in numerating, but the singular form in reading; the word "and" should not be used.

Read:

38. 14635
39. 3785
40. 12005
41. 4607
42. 27815
43. 3010
44. 16001
45. 17000
46. 28001
47. 20000
48. 12070386
49. 5286342
50. 75342786
51. 63000001
52. 500000086
53. 707010060
54. 210805712
55. 11001010
56. 360606080
57. 500000000

Notation — More than One Period

In writing numbers, there are two steps:

I. *The left-hand period is written with one, two, or three figures.*

58. How many figures may there be in the left-hand period? See Ex. 27.

59. Write seven as a left-hand period. *Ans.* 7.

60. Write sixteen as a left-hand period. *Ans.* 16.

61. Write two hundred seven as a left-hand period. *Ans.* 207.

62. Write as a left-hand period: sixty; five; seventeen; twenty; three; seventy-six.

63. Write as a left-hand period: thirty-six; eight; ninety-four; one hundred eight; twenty-three.

II. *The other periods are then written in succession, with three figures each.*

64. How many figures must there be in every period except the left? See Ex. 27.

65. Write seven as a full period. *Ans.* 007.

66. Write sixteen as a full period. *Ans.* 016.

67. Write two hundred seven as a full period. *Ans.* 207.

68. Write naught as a full period. *Ans.* 000.

69. Write as a full period: sixty-one; five; seventy; three; eighty; ninety-one; nine.

70. Write as a full period: thirty-six; nine hundred four; ninety-four; three hundred; one hundred one.

71. Write as a full period: twenty; naught; one; seventeen; thirty-two; one hundred.

Notation — More than One Period

72. Write thirty-five million, one hundred fifty-six.

Ans. 35,000,156. We think *thirty-five million* and write 35; we think *no thousands* and write 000 (the work now appears 35,000); we think *one hundred fifty-six* and write 156 (35,000,156).

Write:

73. Two thousand, eighty-one; one thousand, sixteen.

74. Three hundred five thousand, one hundred ninety-five.

75. Five million, two thousand, eight hundred nine.

76. Twenty-six million, thirty thousand, eighty-seven.

77. Three thousand, three hundred three; four thousand, two.

78. Five hundred million, five hundred thousand, five hundred.

79. Sixty-six million, sixty-six thousand, sixty-six.

80. Five million, five hundred twenty-five thousand.

81. Eighty million, two hundred thousand, sixty-one.

82. Eight million, eight hundred eight thousand, eighty-nine.

83. Six hundred million, six hundred thousand, six hundred six.

84. Forty-two million, nine hundred seventy-six thousand.

85. Three hundred ten million, one hundred two thousand, eighteen.

86. Two hundred twenty million, thirty thousand, seventy-eight.

87. Eighteen million, one hundred fifty thousand, seven hundred thirty.

88. Seven hundred two million, seventy-two thousand, seven hundred twenty.

Roman Notation — United States Money

Roman Notation. — Seven capital letters are used: I, 1; V, 5; X, 10; L, 50; C, 100; D, 500; M, 1000. See p. 28.

89. Write the numbers from XX to XXX. *Ans.* XX, XXI, ...

90. Count by tens to 100, thus: one ten, two tens, three tens; ten from fifty, fifty, fifty and one ten, fifty and two tens, fifty and three tens; ten from one hundred, one hundred.

91. Write by tens to L. *Ans.* X, XX, XXX, XL, L.

92. Write from one to five. *Ans.* I, II, III, IV, V.

93. Compare the answers to examples 91 and 92. Observe that the Romans never use the same letter in succession more than three times: III, then I from V; XXX, then X from L.

94. Write by tens from L to C.

95. Write by Roman numerals all the numbers to 100.

96. This book was written in the year MDCCCXCVIII. Read the date.

United States Money. — The symbol, $, is written before, and a period after, the number of dollars.

97. Write 36 dollars and 53 cents. *Ans.* $36.53.

Write:

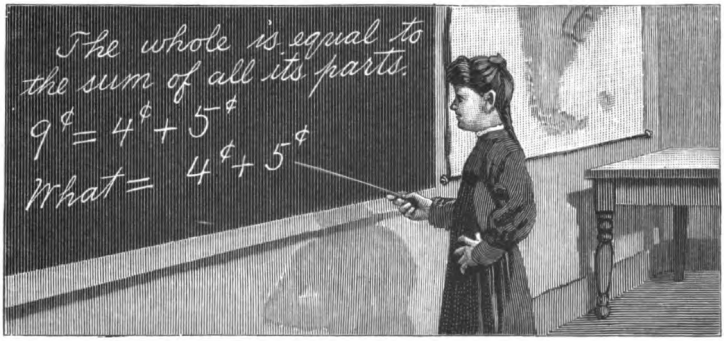
98. 8 dollars 84 cents.
99. 3 dollars 85 cents.
100. 5 dollars 6 cents.
101. 18 dollars 26 cents.
102. 29 dollars 90 cents.
103. 80 dollars 81 cents.

Read:

104. $1.88; $13.50.
105. $9.06; $12.00.
106. $8.28; $72.05.
107. $328.28; $65.03.
108. $302.02; $72.60.
109. $840.40; $83.96.

ADDITION

Terms



Addition is the process of uniting two or more numbers into one.

The numbers to be united are *addends*. The result is the *sum*.

The sign of addition is " +," read *plus;* of equality " =," read *equals*.

ILLUSTRATION. — 3 and 2 are 5; written, 3 + 2 = 5; read, 3 plus 2 equals 5.

1. Prove that 3 + 2 = 5.

Ans. Counting 3, and then 2, and making a mark at each count, we have /// //; counting these together, we have one, two, three, four, five.

2. In "3 + 2 = 5," what are 3 and 2 called? What is 5 called?

Combinations

Addition may be expressed by numbers in columns. Thus, $\begin{smallmatrix}2\\3\end{smallmatrix}$ may mean that the sum of 2 and 3 is to be found.

There are forty-five combinations of the first nine numbers, taken two at a time. They form different ways of expressing numbers from 2 to 18. They are:

1 1, *2*;	2 1, *3*;	2 3 2, 1, *4*;	3 4 2, 1, *5*;	3 4 5 3, 2, 1, *6*;	4 5 6 3, 2, 1, *7*;
4 5 6 7 4, 3, 2, 1, *8*;		5 6 7 8 4, 3, 2, 1, *9*;		5 6 7 8 9 5, 4, 3, 2, 1, *10*;	
6 7 8 9 5, 4, 3, 2, *11*;		6 7 8 9 6, 5, 4, 3, *12*;		7 8 9 6, 5, 4, *13*;	
7 8 9 7, 6, 5, *14*;	8 9 7, 6, *15*;	8 9 8, 7, *16*;	9 8, *17*;	9 9, *18*.	

3. Name the combinations whose sum is 10. *Ans.* 5 6 7 8 9
5, 4, 3, 2, 1.

4. Write 11 in four ways. *Ans.* 6 7 8 9
5, 4, 3, 2.

5. Read: $\begin{smallmatrix}6&9&7&4&8\\3&8&5&9&3\end{smallmatrix}$ *Ans.* 9, 17, 12, 13, 11.

Call the sums rapidly:

	8	3	7	3	6	5	4	1	7	2	1	9	8	6
6.	2	3	2	8	4	1	8	9	9	6	4	8	8	5
	3	9	2	8	6	4	7	6	3	8	5	6	7	9
7.	6	5	1	7	7	9	1	6	1	1	2	8	3	9
	4	8	6	5	9	4	7	6	3	8	5	6	7	9
8.	7	5	1	7	3	4	3	7	6	2	5	5	4	4

Counting by the First Ten Numbers

Count from:

9. 1 to 99 by 2's
10. 1 to 97 by 3's
11. 2 to 98 by 3's
12. 1 to 97 by 4's
13. 2 to 98 by 4's
14. 3 to 99 by 4's
15. 1 to 96 by 5's
16. 2 to 97 by 5's
17. 3 to 98 by 5's
18. 4 to 99 by 5's

Count from:

19. 1 to 97 by 6's
20. 2 to 98 by 6's
21. 4 to 94 by 6's
22. 3 to 99 by 6's
23. 5 to 95 by 6's
24. 6 to 96 by 6's
25. 1 to 99 by 7's
26. 2 to 93 by 7's
27. 3 to 94 by 7's
28. 4 to 95 by 7's

Count from:

29. 5 to 96 by 7's
30. 6 to 97 by 7's
31. 7 to 98 by 7's
32. 1 to 97 by 8's
33. 2 to 98 by 8's
34. 3 to 99 by 8's
35. 4 to 92 by 8's
36. 5 to 93 by 8's
37. 6 to 94 by 8's
38. 7 to 95 by 8's

Count from:

39. 8 to 96 by 8's
40. 1 to 91 by 9's
41. 2 to 92 by 9's
42. 3 to 93 by 9's
43. 4 to 94 by 9's
44. 5 to 95 by 9's
45. 6 to 96 by 9's
46. 7 to 97 by 9's
47. 8 to 98 by 9's
48. 9 to 99 by 9's

Single Columns

Add:

49.	50.	51.	52.	53.	54.	55.	56.
8	2	3	2	6	4	6	4
3	5	4	9	3	8	3	9
4	7	1	1	6	9	6	5

57.	58.	59.	60.	61.	62.	63.	64.
3	9	6	4	6	8	3	4
4	2	3	5	1	4	3	8
2	1	2	3	2	9	7	2
8	7	8	9	9	9	8	8

65.	66.	67.	68.	69.	70.	71.	72.
9	3	7	9	3	7	6	9
4	2	8	9	6	5	8	4
7	4	2	4	8	7	7	3
1	1	7	2	3	4	1	6

73.	74.	75.	76.	77.	78.	79.	80.
8	4	4	2	4	9	2	3
2	2	7	8	1	3	8	4
7	3	2	3	3	8	3	6
3	4	6	9	7	2	4	8
1	2	4	1	5	3	9	2

81.	82.	83.	84.	85.	86.	87.	88.
9	8	8	6	9	7	8	9
8	9	6	7	8	7	8	9
7	3	5	8	9	7	7	8
6	8	9	9	8	6	6	7
5	7	4	3	9	7	8	6

NOTE. — Before solving these examples, the pupil should review pp. 37 and 38.

Problems

89. James caught 9 fish, Henry 8, and Joseph 5. How many fish did they catch in all?

$$\begin{array}{r} 9 \\ 8 \\ 5 \\ \hline 22 \end{array}$$

In all they caught the sum, or 22 fish.

90. A boy with his dog caught 9 rabbits on Monday, 8 on Tuesday, 7 on Wednesday, 6 on Thursday, 5 on Friday, and 9 on Saturday. How many rabbits did he catch during the week?

91. A gardener took to market 9 bushels of potatoes, 8 bushels of onions, and 6 bushels of turnips. How many bushels of produce did he take to market?

92. A boy gathered 8 bushels of apples under one tree, 6 under another, and 3 under another. How many bushels of apples did he gather?

93. There are 9 sheep in one field, 8 in another, 9 in another, and 7 in another. How many sheep are there in all the fields?

94. A man walked 5 miles the first hour, 4 miles the second, 4 miles the third, and 3 miles the fourth. How far did he walk in the four hours?

95. A man rode 1 mile the first hour, 2 miles the second, 3 miles the third, and so on. How far did he go in 9 hours?

96. A man worked one week for as many dollars as there are units in the sum of the first four even numbers. How much did he receive?

97. A boy gave as many cents for a hat as there are units in the sum of the first five odd numbers. How much did he pay for the hat?

Several Columns

98. Add and explain: 28, 36, 25.

2 tens 8 units	*28*
3 tens 6 units	*36*
2 tens 5 units	*25*
8 tens 9 units	*89*

The sum of the units is 19 units, or 1 ten and 9 units; we write 9 in units' column and carry 1 to tens' column.

The sum of the tens is 8 tens; we write 8 in tens' column.

NOTE. — In practice, the work is written as at the right. It is customary to add from the bottom up.

To prove, add the columns from the top down.

99. Prove the answer in Ex. 98.

100. Add and explain: 59, 73, 62, 88, 76.

101. Add and explain: 68, 79, 85, 94, 97, 25, 72, 33, 44.

Copy, add, and prove:

102.	**103.**	**104.**	**105.**	**106.**
89	93	68	478	128
37	47	74	544	456
42	59	87	731	789
59	81	96	397	341
18	26	95	488	678

107.	**108.**	**109.**	**110.**	**111.**
78	63	49	735	654
23	87	10	929	313
56	19	64	648	789
89	75	87	367	357
41	42	35	486	562

Problems

112. A farmer has 96 hens, 54 turkeys, and 14 ducks. How many fowls has he?

96 hens
54 turkeys
14 ducks
164 fowls

He has in all the sum, or 164 fowls.

NOTE. — The sum could not be hens, turkeys, or ducks, but could be fowls, because this is a name common to them all.

113. A man owes \$15 to the butcher, \$12 to the grocer, and \$27 to the baker. How much does he owe in all?

114. Thomas earned \$12 one month, \$9 another month, and \$17 another month. How much did he earn in all?

115. A Christmas tree held 13 dolls, 27 balls, 37 flags, 26 bags, and 21 whistles. How many articles were there?

116. On Monday John had 57¢ in his bank. On Tuesday he earned 25¢; on Wednesday, 42¢; on Thursday, 18¢. If he spent none, how much did he have on Friday?

117. A man had 85 sheep, 69 cows, 14 horses, and 37 pigs. How many animals had he in all?

118. A merchant sold 4 rolls of cloth containing respectively 36, 52, 63, and 29 yards. How many yards of cloth did he sell in all?

119. A horse cost \$39 and was sold for \$12 more than was paid for it. For how much was it sold?

120. In a train of three coaches, the first coach contains 39 passengers; the second, 63; and the third, 74. How many passengers are there on the train?

121. A man had 87 cattle in one lot, 36 in another, 61 in another, and 79 in another. How many cattle were there?

Several Columns

122. Copy, add, and prove: \$48.96, \$73.60, \$49.28, \$10.00.

\$ 48.96	
73.60	*14*
49.28	*18*
10.00	*21*
	18
\$ 181.84	

Be sure to place units of the same kind in the same column.

It is well to write the sum of each column at the right as soon as it is found.

Copy, add, and prove:

123. \$32.86, \$358, \$694, \$73.89, \$247.32, \$843, \$.62, \$.10, \$.89, \$4.38, \$69.49

124. \$65.80, \$80.34, \$914.83, \$678.32, \$146.84, \$264.05, \$104.08, \$320.02, \$200.01, \$468.99

125.	**126.**	**127.**	**128.**
\$4.35	\$5.32	\$84.26	\$17.92
6.21	1.89	59.30	63.14
7.46	6.57	76.52	76.58
1.29	7.00	19.83	45.93
5.83	5.92	40.08	24.70
9.32	3.26	63.94	52.35
4.78	2.87	71.45	37.46
6.57	6.00	28.07	98.67
9.81	8.59	52.63	43.14
3.25	1.73	89.76	15.73
7.46	4.36	24.38	80.94
1.87	5.65	65.72	49.63
3.45	8.76	32.45	78.98
2.60	9.30	45.33	98.22
3.43	7.54	72.45	75.34
2.66	3.65	88.75	80.01

Problems

129. At one time, a man deposited in the bank $375; at another time, $194; at another time, $416. How much had he in the bank in all?

130. A man bought a buggy for $213, and a horse for $175. He sold each for $25 more than it cost him. How much did he receive in all?

131. In one year of school, a college boy paid $125 for tuition, $176 for board, $42 for car fare, and $22 for books. How much were his entire expenses?

132. A lady's hat cost $8.75; her shoes, $5.50; her cloak, $27.37; her dress, $14.28. What was the cost of all?

133. A lady lost a box containing a watch, valued at $75, two rings valued at $85 each, and two chains, valued respectively at $7.50 and $12. What was the amount of her loss?

134. A man's gas bill was $6.38; his coal bill, $8.73; his grocery and meat bill, $12.47 each. How much did he owe?

135. A music dealer bought a piano for $573 and sold it at a gain of $54. What was the selling price?

136. A man shipped 237 head of cattle, 643 hogs, 728 sheep; and then had left 56 head of cattle, 65 hogs, 81 sheep. How many animals had he at first?

137. A man built a house for $4169; the lot cost $1625. He sold the property for $750 more than it cost him. How much did he receive?

138. A capitalist owns 360 acres of land in one state, 547 acres in another, 1753 acres in another, 378 acres in another, and 8 acres in another. How many acres does he own altogether?

Several Columns

In solving each of the following examples, find the answer by adding each column from bottom to top. Then separate into three parts at the places indicated, find the sum of each part separately, and find the sum of the parts; the final result should be the same as the answer first obtained.

139.	140.	141.	142.
21,219	68,304	30,628	8,295
36,453	23,114	9,724	6,685
28,901	64,328	8,685	72,839
46,280	23,265	23,609	64,653
7,329	17,249	62,804	29,945
46,008	3,006	27,300	62,301
30,021	68,329	2,985	7,846
64,278	23,482	33,656	68,394
24,286	19,385	25,381	6,286
16,327	62,178	2,699	48,785
42,006	2,684	3,727	27,345
64,010	96,308	84,347	68,391
9,897	86,486	24,925	8,946
19,328	23,711	65,384	10,109
2,368	62,180	62,198	33,175
46,239	32,608	78,394	86,275
74,206	25,398	6,208	32,609
10,009	84,987	73,401	7,486

The whole sum in Ex. 139 is 589,165

The sum of the first part	186,190
The sum of the second part	240,928
The sum of the third part	162,047
The sum of the parts	589,165

Tests in Speed

	8	6	5	9	8	7	8	6	5	8	5	7
A.	9	7	4	9	7	6	8	6	9	3	6	7.
B.	16	15	18	13	9	16	15	17	18	13	12	10.

143. Read the numbers in B as rapidly as possible.

144. Read the sums in A as rapidly as possible.

Ans. 17, 13, 9, etc.

145. On p. 29 and the following pages you have learned the combinations whose sum is 2, 3, 4, etc., to 18. Can you read these numbers as rapidly in the form expressed in A as in the form in B? If so, you have mastered the combinations.

146. Count by 1's to 25 as rapidly as possible.

147. Count by 9's from 1 to about 100; by 8's; by 7's; by 6's; by 5's; by 4's; by 3's; by 2's.

148. Can you count by each of the numbers below 10 as rapidly as by 1? If so, you have become expert.

149. Add as rapidly as possible: 9, 8, 7, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 7, 8, 9.

150. Add as rapidly as possible: 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1.

151. Can you add the numbers in Ex. 149 as rapidly as those in Ex. 150?

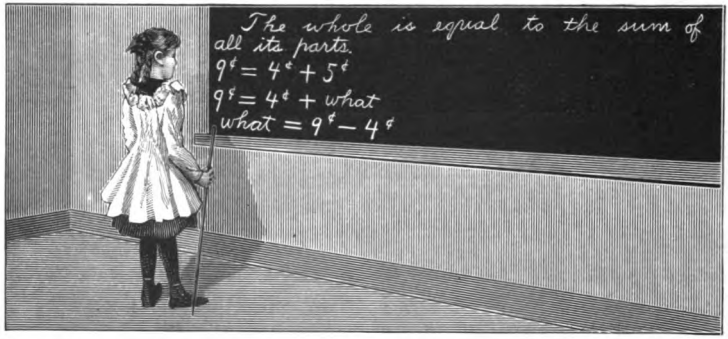
152. Do you not find it harder to keep the place in adding the 1's than in adding the numbers in Ex. 149?

153. Determine how many seconds it takes you to add Ex. 125 on p. 112. If you can find the correct answer in 30 seconds, you are expert.

154. Determine how many seconds it takes you to add Ex. 127 on p. 112. Forty seconds is expert work.

SUBTRACTION

Terms



Subtraction is the process of finding the other of two numbers, when one of them and their sum are given.

The sum is the *minuend;* the number given, the *subtrahend;* the number required, the *difference* or *remainder.*

The sign of subtraction is "−," read minus.

ILLUSTRATION. — 5 less 3 is 2; written, $5 - 3 = 2$; read, 5 minus 3 equals 2.

1. Prove that $5 - 3 = 2$.

Ans. Counting 5 and making a mark at each count, /////; counting 3 and crossing a mark at each count, XXX//; counting what is left, we have 2.

2. In "$5 - 3 = 2$," what is 5 called? What is 3? What is 2?

Combinations

If the forty-five combinations in addition have been mastered, the results in the following examples may be called rapidly. In addition, we have two addends to name the sum; in subtraction, the sum and one addend to name the other.

8

3. ?, *10.* This means, what must be added to 8 to make 10? *Ans.* 2.

Given the sum and one addend, name the other:

1 1 2 3

4. ?, *2*; ?, ?, *3*; ?, *4*?

1 2 1 3

5. ?, ?, *4*; ?, ?, *5*?

2 4 2 5

6. ?, ?, *5*; ?, ?, *6*?

3 4 3 6

7. ?, ?, *6*; ?, ?, *7*?

5 4 2 3

8. ?, ?, ?, *7*; ?, *8*?

2 7 5 4 6

9. ?, ?, ?, ?, ?, *8*?

4 7 5 3 2

10. ?, ?, ?, ?, ?, *9*?

8 6 7 2 9

11. ?, ?, ?, ?, ?, *10*?

3 5 4 8 6

12. ?, ?, ?, ?, ?, *10*?

1 2 6 3

13. ?, *10*; ?, ?, ?, *11*?

7 4 9 5 8

14. ?, ?, ?, ?, ?, *11*?

3 8 4 9 5

15. ?, ?, ?, ?, ?, *12*?

7 6 4 7

16. ?, ?, *12*; ?, ?, *13*?

5 9 6 8

17. ?, ?, ?, ?, *13*?

9 8 7 5

18. ?, ?, ?, *16*; ?, *14*?

8 9 7 6

19. ?, ?, ?, ?, *14*?

6 8 9 7

20. ?, ?, ?, ?, *15*?

9 8 9

21. ?, ?, *17*; ?, *18*?

Each Subtrahend Figure Less

Each figure of the subtrahend may be less than the figure above it in the minuend.

22. From 98 subtract 25, and explain.

9 tens 8 units	*98*
2 tens 5 units	*25*
7 tens 3 units	*73*

IN FULL. — 5 units from 8 units leaves 3 units; we write 3 in units' column. 2 tens from 9 tens leaves 7 tens; we write 7 in tens' column.

ABBREVIATED. — 3, 7. We look at 5 and 8 and say 3; at 2 and 9 and say 7.

NOTE. — In practice, the work is written as at the right.

To prove, add the subtrahend and the remainder. The sum should be the minuend.

23. Prove the answer in Ex. 22. *Ans.* $25 + 73 = 98$.

24. Find the value of $789 - 263$. Write the subtrahend under the minuend and explain in full.

25. Find the value of $963 - 631$. Write the subtrahend under the minuend, and explain as abbreviated.

Copy, subtract, and prove:

26.	**27.**	**28.**	**29.**	**30.**
79	86	974	6768	9657
62	25	312	2507	5306

31.	**32.**	**33.**	**34.**	**35.**
78	58	736	7968	5764
30	34	215	4043	2561

Problems

36. John had 36 marbles and gave away 14 of them. How many had he left?

$$\begin{array}{r} 36 \\ 14 \\ \hline 22 \end{array}$$

He had left the difference between 36 marbles and 14 marbles, or 22 marbles.

37. There were 49 chickens in a yard; 24 were black and the rest were white. How many white ones were there?

38. Jennie's mother is 48 years old, and her sister is 36 years younger. What is her sister's age?

39. James wrote 86 words, but misspelled 12 of them. How many words did he spell correctly?

40. There are 67 passengers in a car. If 32 of them get off at a station, how many of them will remain in the car?

41. Henry has 98¢. If he spends 30¢ for an arithmetic and 43¢ for a reader, how much will he have left?

42. A man had $63; he spent $10 for a stove and $2 for coal. How much money had he left?

43. Charles had 23 problems to solve; he has solved 13 of them. How many more has he to solve?

44. Mary had 27 roses; she gave 13 to Anna and 2 to Ella. How many roses had she left?

45. In an orchard there are 38 apple trees and 16 pear trees. If 11 apple trees are cut down and 6 pear trees are set out, how many more apple trees than pear trees will there be?

46. From a barrel containing 48 pounds of salt, 13 pounds were taken at one time and 22 pounds at another. How many pounds of salt remained?

47. A man's salary for one month was $75; his expenses were $33 for board and $21 for other things. How much money did he save?

One Subtrahend Figure Greater

A figure in the subtrahend may be greater than the figure above it in the minuend.

48. Take 85 splints (8 bundles of ten and 5). Show how you can take away 29 splints (2 bundles of ten and 9).

Ans. Take the band from 1 bundle of ten, leaving 7 bundles of ten; put the 10 single splints and the 5 single splints together, making 15 splints; take away 9 splints; take away 2 tens; you will have left 5 tens and 6 units, or 56.

NOTE. — The pupil should take the splints in his hands and should follow the suggestions.

49. From 85 subtract 29; express the process and explain.

8 tens 5 units	*7 tens 15 units*	*85*
2 tens 9 units	*2 tens 9 units*	*29*
	5 tens 6 units	*56*

IN FULL. — 9 units from 5 units we cannot take; we take 1 ten from 8 tens, leaving 7 tens; 1 ten, or ten units, with 5 units, 15 units; 9 units from 15 units leaves 6 units; we write 6 in units' column.

2 tens from 7 tens leaves 5 tens; we write 5 in tens' column.

ABBREVIATED. — 6, 5. We look upon 5 as 15 and say 6; we look upon 8 as 7 and say 5.

PROOF. — The sum of 29 and 56 is 85.

NOTE. — In practice, the work is written as at the right. It would be a serious mistake for the pupil to form the habit of writing the orders.

50. From 83 subtract 26, explaining the process in full; explaining as abbreviated.

51. From 94 subtract 68, explaining the process in full; explaining as abbreviated.

NOTE. — Before solving these examples, the pupil should review pp. 46, 47, and 48.

Two Columns

Copy, subtract, and prove:

52.	53.	54.	55.	56.	57.	58.
42	50	36	45	90	84	61
27	18	19	28	43	33	19

59.	60.	61.	62.	63.	64.	65.
73	34	62	51	85	91	87
55	16	35	34	47	30	16

66.	67.	68.	69.	70.	71.	72.
30	43	97	86	54	41	64
18	18	28	29	17	18	19

73.	74.	75.	76.	77.	78.	79.
64	93	72	37	48	94	53
27	87	28	19	39	17	28

80.	81.	82.	83.	84.	85.	86.
95	28	54	67	64	58	41
86	19	37	29	25	49	18

Again solve all the examples above without copying. Name each figure of the remainder without writing it, and without trying to state the full answer after it is obtained. Thus: Ex. **52.** Say 5, 1. Ex. **53.** Say 2, 3.

Two or More Subtrahend Figures Greater

Each of several figures in the subtrahend may be greater than the figure above it in the minuend.

87. Take 501 splints (5 bundles of one hundred, and 1). How can you take away 398 splints (3 bundles of one hundred, 9 bundles of ten, and 8)?

Ans. Take the band from 1 bundle of "one hundred," leaving 4 bundles of "one hundred"; take the band from 1 bundle of ten, leaving 9 bundles of ten and 10 singles; put the 10 single splints and the 1 single splint together, making 11 splints; take away 8 splints; take away 9 tens; take away 3 hundreds; you will have left "1 hundred, 0 tens, 3 units," or 103.

88. From 501 subtract 398; express the process and explain.

5 hun. 0 tens 1 unit	*4 hun. 9 tens 11 units*	*501*
3 hun. 9 tens 8 units	*3 hun. 9 tens 8 units*	*398*
	1 hun. 0 tens 3 units	*103*

In full. — 8 units from 1 unit we cannot take; we take 1 hundred from 5 hundreds, leaving 4 hundreds; 1 hundred, or 10 tens, with 0 tens, 10 tens; we take 1 ten from 10 tens, leaving 9 tens; 1 ten, or 10 units, with 1 unit, 11 units.

8 units from 11 units leaves 3 units; we write 3 in units' column; 9 tens from 9 tens leaves 0 tens; we write 0 in tens' column; etc.

Abbreviated. — 3, 0, 1. We look upon 1 as 11, and say 3; we look upon 0 as 9, and say 0; we look upon 5 as 4, and say 1.

Proof. — The sum of 398 and 103 is 501.

Note. — In practice, the work is written as at the right.

89. From 624 subtract 358, explaining the process in full; explaining as abbreviated.

90. From 400 subtract 199, explaining the process in full; explaining as abbreviated.

More than Two Columns

Copy, subtract, and prove:

91.	92.	93.	94.	95.
306	908	506	9203	9382
129	429	227	2614	3967

96.	97.	98.	99.	100.
307	826	921	5891	6306
268	173	365	1792	4307

101.	102.	103.	104.	105.
606	108	627	6206	7000
327	59	168	4187	4391

106.	107.	108.	109.	110.
483	732	574	4625	9273
165	516	364	1232	657

111.	112.	113.	114.	115.
543	839	206	4861	6327
238	479	97	1946	5039

Again solve all the examples above without copying. Name each figure of the remainder without writing it, and without trying to state the full answer after it is obtained. Thus: Ex. **91.** Say 7, 7, 1. Ex. **92.** Say 9, 7, 4.

Problems

116. From a piece of carpeting 73 yards long, a merchant sold 47 yards. How many yards were left?

73, yd. first
47, yd. sold
26, yd. left

He had left the difference, or 26 yards.

117. From a pasture containing 185 cattle, 49 were sold. How many cattle were left in the pasture?

118. A farmer raised 150 bushels of corn. He sold at one time 28 bushels, at another time 46 bushels, and kept the remainder. How many bushels did he keep?

119. On Tuesday a gentleman deposited in a bank $75, on Wednesday he deposited $83, on Thursday he drew out $140. How much did he leave in the bank?

120. A furniture dealer bought some chairs for $9.50; it cost him $3.75 to varnish them. He sold them for $15. How much did he gain?

121. A man bought a load of coal for $4.45, and a ton of hay for $3.25. He gave in payment a ten-dollar bill. How much change did he receive?

122. A man owning 365 acres of land, purchased a tract of 240 acres, and then sold 320 acres. How many acres did he have left?

123. A farmer having 263 cattle, bought 847 more. He then shipped all but 359. How many did he ship?

124. The sum of three numbers is 873. The first is 138, the second is 256. What is the third?

125. Two trains start from the same station. One runs 736 miles east, the other 1085 miles west. How far apart are the two trains?

Problems

126. If a man earns $275, he will be able to pay a debt of $2743. How much money has he now?

127. A newspaper has 3670 subscribers, 385 of whom live in the country. How many live in the city?

128. A man borrowed 2 fifty-dollar bills, 2 twenty-dollar bills, 6 ten-dollar bills, and 5 five-dollar bills. After paying back $79.29, how much did he still owe?

129. Two cities, A and B, are 789 miles apart. A train starts from A and runs 235 miles toward B; another train starts from B and runs 146 miles towards A. How far apart are the two trains?

130. A man's income is $3800 a year. He pays $375 a year for house rent, $175 for taxes, $280 for hired help, and $560 for other expenses. How much are his total expenses? How much has he left each year?

131. From seven thousand sixty-four, subtract three thousand eight hundred seven; then subtract the difference between two thousand and one thousand one.

132. The greater of two numbers is 8027, and their difference is 3842. What is the smaller number?

133. A man paid $3265 for a house and lot; he paid $2950 for the house. How much did he pay for the lot?

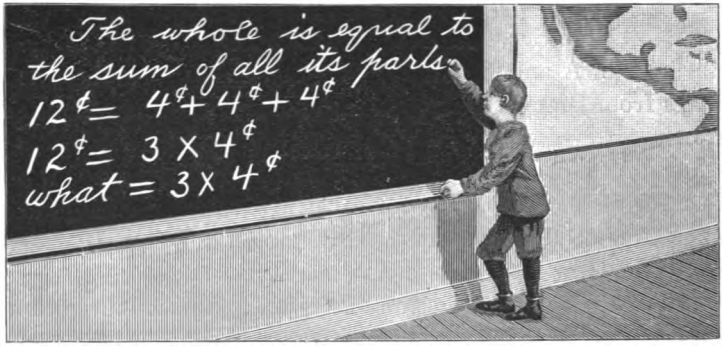
134. What number is 2857 less than 9264?

135. In a library of 7490 volumes, 875 are histories, 356 books of science, 1092 biographies, and the rest books of fiction. How many books of fiction are there?

136. The face of a clock is numbered from I to XII. If it were numbered from I to XXIV, what would be the difference between the sum of the numbers in the one case and in the other?

MULTIPLICATION

Terms



Multiplication is the process of finding the sum when the same number is used several times as an addend.

The number used as an addend is the *multiplicand;* the number showing how many times, the *multiplier;* the result, the *product;* the terms, *factors.*

The sign of multiplication is " ×," read *times* when it precedes the multiplicand; read *multiplied by* when it follows the multiplicand.

ILLUSTRATION. — $4¢ + 4¢ + 4¢ = 3 \times 4¢$, or $4¢ \times 3$. In each case, $4¢$ is the multiplicand; 3, the multiplier.

"$3 \times 4¢$" is read *3 times 4 cents;* "4 cents × 3" is read *4 cents multiplied by 3;* "3×4" is read *3 times 4*, or *4 multiplied by 3*, the former being preferred.

Exercise in the Tables

The combinations in multiplication to "9 × 9" have been given on pages 63 and 65. They should be thoroughly reviewed.

1. Make and memorize the table of "11 times" to 11 × 9.

11 × 1 = 11	11 × 4 = 44	11 × 7 = 77
11 × 2 = 22	11 × 5 = 55	11 × 8 = 88
11 × 3 = 33	11 × 6 = 66	11 × 9 = 99

2. Make and memorize the table of "12 times" to 12 × 9.

12 × 1 = 12	12 × 4 = 48	12 × 7 = 84
12 × 2 = 24	12 × 5 = 60	12 × 8 = 96
12 × 3 = 36	12 × 6 = 72	12 × 9 = 108

3. How many are: 12 × 9? 12 × 3? 12 × 7? 12 × 4? 12 × 2? 12 × 5? 12 × 8? 12 × 1? 12 × 6?

9, 2, 7, 6, 3, 8, 5, 1, 4:

4. Multiply by 2 and add 1. *Ans.* 19, 5, 15, 13, 7, 17, 11, 3, 9.

5. Multiply: by 3 and add 1; by 3 and add 2.

6. Multiply: by 4 and add 1; by 4 and add 2; by 4 and add 3.

7. Multiply: by 5 and add 1; add 2; add 3; add 4.

8. Multiply: by 6 and add 1; add 2; add 3; add 4; add 5.

9. Multiply: by 7 and add 1; add 2; add 3; add 4; add 5; add 6.

10. Multiply: by 8 and add 1; 2; 3; 4; 5; 6; 7.

11. Multiply: by 9 and add 1; 2; 3; 4; 5; 6; 7; 8.

12. Multiply: by 11 and add 1; 2; 3; 4; 5; 6; 7; 8; 9.

13. Multiply: by 12 and add 1; 2; 3; 4; 5; 6; 7; 8; 9; 10.

Multiplier Less than Thirteen

14. Multiply 136 by 6 and explain.

1 hundred 3 tens 6 units	*136*
6	*6*
8 hundreds 1 ten 6 units	*816*

IN FULL. — 6 times 6 units are 36 units, or 3 tens and 6 units; we write 6 in units' column and carry 3 tens.

6 times 3 tens are 18 tens, with 3 tens, 21 tens or 2 hundreds and 1 ten; we write 1 in tens' column and carry 2 hundreds.

6 times 1 hundred are 6 hundreds, with 2 hundreds, 8 hundreds; we write 8 in hundreds' column.

ABBREVIATED. — 36, 21, 8.

NOTE. — In practice, the work is written as at the right.

To prove, go over the work a second time.

Multiply and prove, explaining in full:

15.	**16.**	**17.**	**18.**	**19.**
28	56	126	448	396
2	3	2	4	6
20.	**21.**	**22.**	**23.**	**24.**
29	58	137	459	983
3	4	6	7	5
25.	**26.**	**27.**	**28.**	**29.**
57	63	896	903	634
12	11	9	8	7

Multiplier Less than Thirteen

Copy, multiply, and prove, abbreviating:

30.	31.	32.	33.
1804	32,986	98,065	240,721
9	5	8	8

34.	35.	36.	37.
7608	87,063	47,308	706,204
8	7	3	9

38.	39.	40.	41.
6024	43,970	20,109	903,026
12	6	11	7

42.	43.	44.	45.
5160	25,709	13,876	263,876
5	11	12	8

46.	47.	48.	49.
4829	96,430	10,963	560,721
5	11	12	8

Again solve all the examples above without copying. Name each figure of the remainder without writing it, and without trying to state the full answer after it is obtained. Thus: Ex. 30. Say 36, 72, 16. Ex. 31. Say 30, 43, 49, 14, 16.

Problems

50. A farmer bought 9 carloads of cattle, averaging 27 to a car. How many cattle did he purchase?

27, in one car
9
243, in all

In all, he bought 9 times 27 cattle, or 243 cattle.

51. Four men doing business in partnership find each one's profit, at the end of the year, to be $2689. What is the profit of the entire firm?

52. In a pamphlet of 10 pages, there are 12 lines to the page and 12 words to the line. How many words are there in the whole pamphlet?

53. A grocer bought 125 pounds of coffee at 12¢ per pound, 98 pounds of sugar at 8¢ per pound, 75 pounds of crackers at 6¢ per pound, and 102 pounds of starch at 7¢ per pound. How much did his entire purchase cost him?

54. A takes a railway journey of 58 miles; B goes 6 times as far; C, 5 times as far as A and B together; D, 4 times as far as B. How far do all travel?

55. A farmer's wife sells 4 dozen hens, weighing 4 pounds each, at 4¢ a pound; 7 dozen turkeys, weighing 12 pounds each, at 7¢ a pound; and 28 pounds of butter at 11¢ a pound. How much does she receive in all?

56. Sound travels about 1189 feet per second. If the stroke of a woodman's ax is heard 3 seconds after the ax is seen to strike, how far distant is the woodman?

57. A contractor agrees to build 8 houses at $1289; and 4 other houses at $6789 each. How much is his gain if he pays for labor $8267, and for lumber and other materials $17,268?

Multiplier Greater than Twelve

58. Multiply 75 by 36 and explain.

75	*75 × 6 units = 450 units*
36	*75 × 3 tens = 2250 units (225 tens)*
450	*75 × 36 = 2700 units*
225	We place the right-hand figure of each partial product
2700	under that figure of the multiplier which produces it.

NOTE. — It is customary to begin to multiply by units' figure of the multiplier.

Perform the indicated operation:

59. 24 × 37	**62.** 238 × 496
60. 48 × 56	**63.** 457 × 243
61. 49 × 38	**64.** 593 × 268

65. Multiply 7205 by 3002.

7205	
3002	If there are ciphers within the multiplier, we are
14410	careful to place the right-hand figure of each partial
21615	product under that figure of the multiplier which
21629410	produces it.

Perform the indicated operation:

66. 3608 × 2004	**70.** 2008 × 3406
67. 2962 × 2032	**71.** 3086 × 2876
68. 4856 × 2102	**72.** 4293 × 7207
69. 4084 × 2405	**73.** 8098 × 7009

Multiplier Greater than Twelve

74. Multiply 234 by 100.

$$\begin{array}{r} 234 \\ 100 \\ \hline 23400 \end{array}$$

If the multiplier is 10, we annex one cipher to the multiplicand; if 100, two ciphers; and so on.

Perform the indicated operation:

75.	349×100	**80.**	8306×100
76.	603×10	**81.**	9002×10
77.	736×1000	**82.**	7389×1000
78.	368×10	**83.**	8306×1000
79.	476×1000	**84.**	7000×1000

85. Multiply 2300 by 240.

$$\begin{array}{r} 2300 \\ 240 \\ \hline 92 \\ 46 \\ \hline 552000 \end{array}$$

If one or both terms end with ciphers, we neglect the ciphers in multiplying, and annex to the product as many ciphers as have been neglected.

Perform the indicated operation:

86.	3600×200	**91.**	3027×3100
87.	3070×250	**92.**	6002×7080
88.	4560×360	**93.**	7009×9030
89.	3700×480	**94.**	6007×8000
90.	9000×400	**95.**	9001×6000

Multiplier Greater than Twelve

Perform the indicated operation:

96. 234×25
97. 520×32
98. 174×41
99. 462×23
100. 546×54
101. 257×16
102. 503×67
103. 463×46
104. 736×29
105. 120×78
106. 563×70
107. 215×87

Perform the indicated operation:

108. 6785×76
109. 4009×80
110. 9708×45
111. 7093×67
112. 5169×75
113. 8706×63
114. 8496×76
115. 7085×69
116. 5876×94
117. 9876×58
118. 8027×89
119. 5409×67

Perform the indicated operation:

120. 4635×213
121. 5762×451
122. 1067×603
123. 2408×723
124. 7450×106
125. 2435×705
126. 6008×5736
127. 4609×8097
128. 9028×7070
129. 9876×9780
130. 7608×5700
131. 3009×6930

Problems

132. At $2.36 each, how much will 6 pigs cost?

$2.36, cost of 1 pig
6
$14.16, cost of all

6 pigs will cost 6 times $2.36, or $14.16.

NOTE. — We multiply 236 by 6 as in integers and point off 2 figures for cents.

133. At a factory, 273 chairs are made each working day at a cost of $1.27 each. What is the daily expense?

134. In a certain cornfield there are 735 rows, and 128 hills in each row. How many hills are there in the field?

135. A farmer sold 642 bales of hay weighing 280 pounds each. How many pounds did he sell?

136. If a man's salary is $867.55 a year, what will it be for 13 years?

137. A man divided his property among his four children, giving to each $387.59. What was the value of his estate?

138. Thomas is 6 years old. John is 3 times as old as Thomas, and their mother is 3 times as old as John. How many years in their combined ages?

139. A jeweler bought 3 dozen rings at $18 each, and 2 dozen watches at $35 each. How much did they cost him?

140. A miller bought 7 loads of barrels, each containing 146 barrels, at 28¢ each. How much did they cost him?

141 An army used 8654 pounds of meat each month. How much did they use in two years?

142 A farmer raised 65 acres of corn which yielded 23 bushels to the acre. He sold it for 18¢ a bushel. How much did he receive?

Problems

143. A man paid 17¢ each for 15 chickens, and 2¢ each for two dozen eggs. How much did he pay altogether?

144. A man earns $28 a week, and his son earns $16 a week. How much will they both earn in 12 weeks?

145. A man's salary is $175 per month, and his monthly expenses are $68. How much does he save in 14 years?

146. A furniture dealer bought one half dozen chairs at $1.25 each, 3 tables at $9 each, and 4 bedsteads at $15 each. What was his bill?

147. A lady bought 23 yards of muslin at 8¢ a yard, 25 yards of carpeting at $1.40 a yard, and 12 yards of dress goods at $2.13 a yard. She gave in payment a 50-dollar and a 20-dollar bill. How much change should she receive?

148. A salesman sold 3 lots of goods. For the first he received $985; for the second, twice as much as for the first; and for the third, 3 times as much as for the other two. How much did he receive for all?

149. The annual expense of a school for 5 years, exclusive of teachers' salaries, was $2968.75. In the school were 12 teachers whose average annual salary was $810.50. What was the total annual expense of the school?

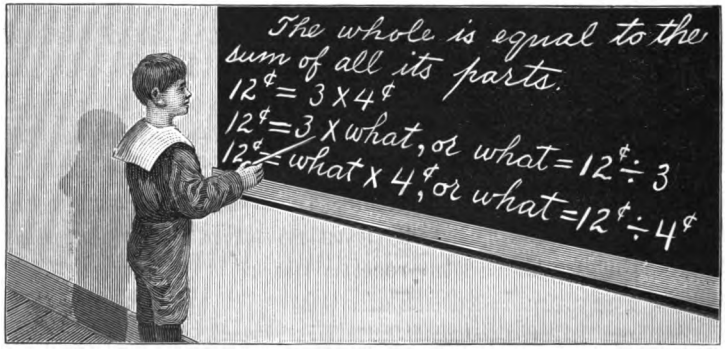
150. When corn is 47¢ a bushel, and wheat is 89¢ a bushel, how much more will 72 bushels of corn cost than 36 bushels of wheat?

151. There are 28 kegs of nails, each weighing 136 pounds, and the kegs which contain the nails weigh 7 pounds each. What is the weight of the nails without the kegs?

152. A man bought 14 barrels of apples at $3.75 a barrel, and gave in exchange 6 sacks of flour at $1.50. How much does he still owe?

DIVISION

Terms



Division is the process of finding the other of two numbers, when one of them and their product are given.

The product is the *dividend;* the number given, the *divisor;* the number required, the *quotient.*

The sign of division is " ÷ " ; read *divided by.*

ILLUSTRATION. — "12¢ = 3 × what" means "What number must be used 3 times as an addend to produce 12¢?" It is commonly written, "*What* = *12¢ ÷ 3¢?*" and read, "*What equals 12¢ divided by 3¢?*"

"12¢ = what × 4¢" means "How many times must 4¢ be used as an addend to produce 12¢?" It is commonly written, "*What* = *12¢ ÷ 4¢?*" and read, "*What equals 12¢ divided by 4¢?*"

Combinations

If the multiplication table has been mastered, the results in the following examples may be called rapidly. In multiplication, we have two factors to name their product; in division, the product and one factor to name the other.

1. Given the product and one factor, name the other.

9 12
?, ?, *72.* *Ans.* 8, 6; because $9 \times 8 = 72$, $12 \times 6 = 72$.

Given the product and one factor, name the other:

	12	12	2 3 4	3 4 6	4 6	
2.	?, *96*;	?, *108*;	?, ?, ?, *24*;	?, ?, ?, *36*;	?, ?, *48*.	
	8	12 10	6 8	7	9	10
3.	?, *96*;	?, ?, *60*;	?, ?, *72*;	?, *84*;	?, *108*;	?, *90*.
	5	6	11	9	8	3	10
4.	?, *55*;	?, *66*;	?, *121*;	?, *99*;	?, *88*;	?, *33*;	?, *80*.
	2	7	4	3	10	2 3	5
5.	?, *22*;	?, *77*;	?, *44*;	?, *27*;	?, *70*;	?, ?, *18*;	?, *45*.
	9	7	6	8	8	8	7	7
6.	?, *81*;	?, *63*;	?, *54*;	?, *64*;	?, *72*;	?, *80*;	?, *49*;	?, *70*.
	2	3	4	5	6	7	7	4
7.	?, *14*;	?, *21*;	?, *28*;	?, *35*;	?, *42*;	?, *56*;	?, *70*;	?, *40*.
	3 5	2 3	2 4	4	5	3
8.	?, ?, *30*;	?, ?, *12*;	?, ?, *16*;	?, *32*;	?, *50*;	?, *15*.
	2 4	5	2	2	3	2	2
9.	?, ?, *20*;	?, *25*;	?, *10*;	?, *18*;	?, *18*;	?, *16*;	?, *12*.

The Remainder

The dividend is not always the product of the divisor and an integer.

In this case, the largest integral quotient is found, and the remainder, obtained by subtracting the product from the dividend, is left undivided. The remainder is usually written above and the divisor below a horizontal line, to show that the division has not been performed. Thus:

In $11 \div 5$, the largest integral quotient is 2, because $2 \times 5 = 10$, and 10 from 11 leaves 1, a remainder smaller than the divisor. Therefore $11 \div 5 = 2$ with a remainder, 1, which is left undivided; or $11 \div 5 = 2\frac{1}{5}$; read, $11 \div 5 = 2$ and $1 \div 5$, or two and one fifth.

Find the value of:

10. $18 \div 4$, $13 \div 6$, $25 \div 4$, $37 \div 9$, $17 \div 5$, $16 \div 7$, $26 \div 5$, $21 \div 4$, $32 \div 5$, $26 \div 8$, $29 \div 2$.

11. $33 \div 6$, $29 \div 2$, $38 \div 3$, $23 \div 2$, $27 \div 4$, $25 \div 4$, $23 \div 7$, $22 \div 5$, $19 \div 9$, $28 \div 8$, $21 \div 2$.

12. $16 \div 6$, $19 \div 8$, $15 \div 7$, $13 \div 5$, $19 \div 3$, $33 \div 4$, $50 \div 7$, $64 \div 9$, $28 \div 3$, $109 \div 9$, $97 \div 8$.

Ans. Ex. **10.** 4, 2; 2, 1; 6, 1; . . . .

Find the value of:

13. 112, 99, 86, 75, 63, 49, 38, 27, 13, 53, 29, $\div 12$.

14. 100, 89, 81, 79, 67, 58, 46, 39, 28, 30, 60, $\div 11$.

15. 110, 83, 93, 85, 73, 66, 59, 48, 28, 42, 39, $\div 9$.

16. 97, 89, 76, 71, 59, 49, 34, 27, 19, 8, 70, 60, $\div 8$.

Ans. Ex. **13.** 9, 4; 8, 3; 7, 2; . . . .

Problems

17. If a coat can be made from 3 yards of cloth, how many such coats can be made from 19 yards?

3)*19*, *yards in all*
6, *no. coats*
1, *no. yards left*

As many coats can be made from 19 yards, as 3 yards is contained times in 19 yards, or 6 coats. 1 yard will be left.

18. At $9 each, how many calves must a man sell to pay a debt of $25? How much will he have left after paying the debt?

19. When butter is 9 cents a pound, how many pounds can be bought for 67 cents?

20. Edward's father gives him 28 cents to buy firecrackers for the Fourth of July. How many bunches can he buy at 3 cents a bunch?

21. If eggs sell for 8 cents a dozen, how many dozen can I buy for 79 cents? How many apples at 3 cents apiece can I buy with the money remaining? How much money will I have left?

22. If one square of oilcloth will cover 9 square feet, how many squares will it take to cover a floor which contains 111 square feet?

23. It costs a furniture dealer $19 to have a lot of chairs varnished, at $2 a dozen. How many dozen chairs are there? How many chairs?

24. A farmer desires to place 35 pigs in 4 pens. Can he put the same number in each pen?

25. He decides to put 9 pigs in each of three pens. How many must be put in the fourth?

26. How can a man divide 28¢ among 8 children, and give each as nearly as possible the same sum of money?

Short Division

27. Divide 263 by 3.

3)26 tens 3 units	*3)263*
8 tens 7 units $\frac{2}{3}$	*87* $\frac{2}{3}$

IN FULL. — 26 tens ÷ 3 = 8 tens and 2 tens remaining; we write 8 in tens' column.

2 tens and 3 units = 23 units; 23 units ÷ 3 = 7 units and 2 units remaining; we write 7 in units' column and 3 under 2, with a line between, to show that 2 is still to be divided by 3.

ABBREVIATED. — 8, 7, and $\frac{2}{3}$.

NOTE. — In practice, the work is written as at the right.

PROOF.

$87\frac{2}{3}$
3
2
261
263

If 263 ÷ 3 = 87 with a remainder of 2, then 87 × 3 + 2 must equal 263.

To prove, multiply the divisor by the quotient and add the remainder. The result should be the dividend.

Divide and prove, explaining in full:

28.	**29.**	**30.**	**31.**	**32.**
3)134	2)375	4)496	5)625	8)864
33.	**34.**	**35.**	**36.**	**37.**
12)496	11)378	7)965	8)384	9)576

Short Division

Copy, divide, and prove:

38.	39.	40.	41.
12)384	12)4704	11)35861	9)23868
42.	**43.**	**44.**	**45.**
8)6864	7)7835	9)9828	11)98124
46.	**47.**	**48.**	**49.**
6)736501	8)937682	7)436944	12)396483
50.	**51.**	**52.**	**53.**
7)686354	5)386555	4)736522	6)938645
54.	**55.**	**56.**	**57.**
4)9365843	6)9285009	3)8698233	7)869266
58.	**59.**	**60.**	**61.**
9)1080544	8)12809203	11)1540888	12)1704036

Again solve all the problems above without copying. Name each figure of the remainder without writing it, and without stating the full answer after it is obtained. Thus:

Ex. 38. Say 3, 2. **Ex. 39.** Say 3, 9, 2.

Long Division

62. Divide 304 by 13.

$$\begin{array}{r} 23\tfrac{5}{13} \\ 13\overline{)304} \\ \underline{26} \\ 44 \\ \underline{39} \\ 5 \end{array}$$

30 tens ÷ 13 = 2 tens and 4 tens remaining; we write 2 in tens' column.

4 tens and 4 units = 44 units; 44 units ÷ 13 = 3 units and 5 units remaining; we write 3 in units' column and 13 under 5 with a line between, to show that 5 is still to be divided by 13.

Divide and explain:

63. 390 ÷ 15

64. 448 ÷ 16

65. 972 ÷ 18

66. 966 ÷ 14

67. 959 ÷ 17

68. 897 ÷ 13

69. 817 ÷ 19

70. 869 ÷ 18

71. Divide 452 by 113.

$$\begin{array}{r} 4 \\ 113\overline{)452} \\ \underline{452} \end{array}$$

To find the quotient figure when the first figure of the dividend is larger than the first figure of the divisor, we use only the first figure of each term.

Approximately, 452 ÷ 113 = 4 ÷ 1.

NOTE. — It often happens, as in Ex. 72, that the figure of the divisor thus obtained is too large.

Perform the indicated operation:

72. 805 ÷ 115

73. 791 ÷ 113

74. 696 ÷ 116

75. 424 ÷ 212

76. 8124 ÷ 219

77. 6322 ÷ 218

78. 9699 ÷ 334

79. 8648 ÷ 376

Problems

80. A grocer had 1296 eggs in 54 boxes, with the same number in each. How many dozen had he in each box?

```
54)1296(24
  108
  216
  216
```

In 1 box he had $\frac{1}{54}$ of 1296 eggs, or 24 eggs, or 2 dozen eggs.

81. How many tons of hay at $17 a ton must be given for 18 cows worth $34 each?

```
17)34(2
  34
```

One cow is worth 2 tons of hay; 18 cows are worth 36 tons of hay.

82. A man paid $273 for 13 horses. What was the price of each horse?

83. At $24 each, how many cows can be bought for $384?

84. If $3740 is divided equally among 17 children, how much will each receive?

85. A pipe discharges 72 gallons in an hour. In how many hours will it empty a cistern containing 6912 gallons?

86. A farmer sold 39 calves at the rate of 16 for $128. How much did he receive for them?

87. An errand boy earns $18 a month, and spends $4 a month. In how many months will he save $182?

88. If 234 bushels of grain fill 13 bins, how many bushels will be required to fill 24 bins of equal size?

89. If a man travels 2016 miles in 18 days, how far will he travel in 15 days?

90. How many weeks are there in 6216 hours?

91. A bushel of corn weighs 56 pounds. How many bushels in 29,808 pounds of corn?

Long Division

92. Divide 1652 by 236.

$$\begin{array}{r} 7 \\ \hline 236)1652 \\ 1652 \\ \hline \end{array}$$

To find the quotient figure when the first figure of the dividend is smaller than the first figure of the divisor, we use only the first two figures of the dividend and the first figure of the divisor.

Approximately, $1652 \div 236 = 16 \div 2$, or 8. $8 \times 236 = 1888$. Since 8 is too large, we try 7.

Perform the indicated operation:

93. $3234 \div 462$

94. $2376 \div 396$

95. $3912 \div 489$

96. $2528 \div 632$

97. $5481 \div 783$

98. $2583 \div 369$

99. $2916 \div 729$

100. $4122 \div 687$

101. Divide 2154 by 359.

$$\begin{array}{r} 6 \\ \hline 359)2154 \\ 2154 \\ \hline \end{array}$$

To determine whether the quotient figure is too large, it is rarely necessary to use more than the first two figures of the divisor.

Approximately, $2154 \div 359 = 21 \div 3$, or 7. $7 \times 35 = 245$. Since 7 is too large, we try 6.

Perform the indicated operation:

102. $5486 \div 682$

103. $6392 \div 799$

104. $3168 \div 396$

105. $2637 \div 293$

106. $34082 \div 4869$

107. $22536 \div 3756$

108. $28392 \div 4732$

109. $27692 \div 3956$

Problems

110. If a man's yearly income, received in monthly installments, is $2820, how much does he receive in 8 months?

111. If 27 bushels of grain fill 3 bins, how many bins will be required to hold 378 bushels?

112. If a barrel of flour weighs 196 pounds, how many poor families can be supplied from 25 barrels of flour, allowing 25 pounds to each family?

113. If 4 men can do a piece of work in 24 days, in how many days should 6 men be able to do the same work?

114. At the rate of 7 oranges for 63¢, what will be the cost of 3 dozen oranges?

115. A man earns $75 a month, and spends $5 a week for board, and $3 for other expenses. In how many months will he save $516? Count 4 weeks as a month.

116. In a book of 320 pages, 34 lines to the page, there are 533,120 letters. How many letters are there in each line?

117. A merchant's sales in 2 years amounted to $348,936. What were his average sales per month?

118. A carpenter bought lumber to the amount of $18,765. He paid $2991 in cash, and the remainder in 33 equal payments. What was the amount of each payment?

119. Divide eight million two hundred seventy thousand nine hundred fifty-five, by nine thousand seven hundred sixty-five.

120. Divide eighty-six million three hundred four thousand six, by three hundred sixty-five.

121. A farmer raised 6578 bushels of corn; he sold 520 bushels to one man, 238 bushels to another, and fed the remainder at the rate of 97 bushels per day. How many days did the corn last?

Long Division

122. Divide 722437 by 893.

$893)722437(809$
$\underline{7144}$
8037
$\underline{8037}$

$72 \div 8 = 9$, $9 \times 89 = 801$. Since 9 is too large, we try 8.

Since the product of 893 by 8 must contain four figures, we place 4, the right-hand figure of the product, under the fourth figure of the dividend.

NOTE. — The quotient is often written as above.

123. Divide 181,076 by 892.

124. Divide 7384 by 100.

$100)7384$
$73\frac{84}{100}$

If the divisor is 10, we cut off one figure from the right of the dividend; if 100, two figures; and so on.

125. Divide 32078 by 1000.

126. Divide 32187 by 9200.

$92.00\times)321.87\times(3\frac{4587}{9200}$
$\underline{276}$
45

If the divisor ends with ciphers, we cut off the ciphers and the same number of figures from the right of the dividend.

We divide the parts left and prefix the remainder to the part cut off, to find the true remainder.

NOTE. — A cross may be placed after units, and a point before the last figure cut off.

127. Divide 280765 by 37000.

128. Prove the answer to Ex. 126.

$3\frac{4587}{9200}$
9200
$\underline{4587}$
27600
$\underline{32187}$

Multiplying the quotient by the divisor and adding the remainder, we obtain the dividend.

Problems

129. A man owes a debt amounting to $288. If he pays $72 in 4 months, how long a time will it take him to pay it all at that rate?

130. The wages of a school teacher are $25 per week, and his expenses are $6 per week. In how many months can he pay a debt of $176 and deposit $185 in the bank? Count 4 weeks as a month.

131. The expense of shipping horses to Chicago was $263. If the entire expense had been $8 less, the expense would have been $17 for each horse. How many horses were shipped?

132. The cost of 320 acres of land, at $65 an acre, was $8580 more than the cost of 13 city lots. What was the cost of each lot?

133. A drover, with $2142, bought as many horses as possible for $15 each, and spent the remainder for calves at $4 each. How many of each did he buy?

134. A leases a house for $35 per month, and the gas bill for the house costs him $8 per month. In how many months will the house cost him $688?

135. Into how many farms of 117 acres each can a tract of land containing 4563 acres be divided?

136. A merchant makes this year in business $1873. This is $207 more than 14 times as much as he was worth three years ago. How much was he worth three years ago?

137. There are 5280 feet in a mile. How many miles in 26,400 feet?

138. In one day the output of a mill is 26,950 pounds of flour. If this is put into sacks containing 49 pounds each, how many sackfuls will there be?

Long Division

Perform the indicated operation:

139. $368 \div 16$

140. $990 \div 22$

141. $294 \div 14$

142. $2457 \div 27$

143. $1955 \div 23$

144. $18791 \div 43$

145. $88504 \div 184$

146. $75077 \div 77$

147. $85176 \div 168$

148. $49179 \div 507$

149. $54054 \div 91$

150. $68952 \div 221$

Perform the indicated operation:

151. $8750 \div 100$

152. $9675 \div 711$

153. $9000 \div 1000$

154. $7200 \div 100$

155. $69120 \div 480$

156. $45600 \div 100$

157. $55000 \div 550$

158. $95000 \div 1900$

159. $472500 \div 630$

160. $32400 \div 320$

161. $300000 \div 250$

162. $852000 \div 3000$

Perform the indicated operation:

163. $38502 \div 207$

164. $87726 \div 338$

165. $450000 \div 1000$

166. $44640 \div 240$

167. $987650 \div 4600$

168. $194556 \div 523$

169. $49410 \div 162$

170. $885715 \div 235$

171. $230045 \div 1000$

172. $253134 \div 809$

173. $130500 \div 900$

174. $361920 \div 604$

Problems

175. If the cost of constructing 457 miles of railway is $2,003,945, what is the cost of construction per mile?

176. If a plot of ground containing 17 acres is cut up into 34 equal lots, how many square rods will there be in each lot? (160 square rods equal 1 acre.)

177. My farm is 86 rods long and 72 rods wide. How much will it cost to fence it at $.47 a rod?

178. A man sells 2 farms for $1227 each. The expenses of each sale are $127. With the proceeds, he buys horses at $55 each. How many horses does he buy?

179. A grocer sells 12 pounds coffee at 15¢; 7 pounds tea at 42¢; 9 pounds crackers at 9¢; 5 cans peaches at 28¢; 36 pounds sugar at 6¢; 4 sacks of flour at $1.12; and takes in exchange 40 pounds butter at 9¢; 4 dozen chickens, averaging 2 pounds each, at 9¢ a pound; 24 dozen eggs at 8¢. How much does the grocer still owe?

180. A's farm of 596 acres is divided into 4 equal parts; of these parts, one is in wheat, one is in oats, and the remaining two in corn. His corn yield for a certain year is 12,516 bushels; the wheat yield, 4172 bushels; and the oat yield, 6505 bushels. What is the yield of each crop per acre?

181. In the above problem, if A markets his corn at 22¢, his wheat at 61¢, and his oats at 26¢, how much will he receive for his entire crop?

182. Three miners sell their claim for $33,000. This sum lacks $1731 of being 17 times the original capital of the first miner; is $2880 more than 20 times the original capital of the second miner; and is $18,000 more than 15 times the original capital of the third. How much were the three miners together worth when they entered partnership?

Factors

A *factor* of a number is an exact divisor of that number.

183. Find a factor of 12. *Ans.* 2; 2 is an exact divisor of 12.

184. Find all the factors of 12.

Ans. 1, 2, 3, 4, 6, 12; each of these numbers is an exact divisor of 12.

185. Find all the factors: of 1; of 2; of 3; of 4; of 5; of 6; of 7; of 8; of 9; of 10; of 11.

186. Find all the factors: of 13; of 14; of 15; of 16; of 17; of 18; of 19; of 20; of 21; of 22; of 23.

187. Find all the factors: of 24; of 25; of 26; of 27; of 28; of 29; of 30; of 31; of 32; of 33; of 34; of 35.

188. Find all the factors: of 36; of 37; of 38; of 39; of 40; of 41; of 42; of 43; of 44; of 45; of 46; of 47.

189. Is there any number that does not have itself and 1 as factors?

190. Write each number from 1 to 20 that has other factors besides itself and 1. All such are *composite* numbers.

191. Write the composite numbers from 21 to 50.

192. Write each number from 1 to 20 that has no other factors besides itself and 1. All such are *prime* numbers.

193. Write the prime numbers from 21 to 50.

194. Is 71 exactly divisible by 2? by 3? by 4? by 5? by 6? by 7? by 8? by 9?

195. Is 71 a prime number? How do you know?

SUGGESTION. — See if 2, 3, 4, 5, 6, 7, 8, or 9 is an exact divisor. Try numbers larger than 9, and see if you can find any reason why this is not necessary.

196. Write the prime numbers from 51 to 100.

197. Is 101 a prime number? Is 111 a prime number?

Greatest Common Factor

A *common factor* of two or more numbers is an exact divisor of each number.

198. Find a common factor of 6 and 8.

Ans. 2; 2 is an exact divisor of each.

199. Find all the common factors of 12, 18, and 24.

Ans. 1, 2, 3, 6; each is an exact divisor of 12, of 18, and of 24.

200. Find the greatest common divisor of 15, 30, and 45.

Ans. 15. The common factors are: 1, 3, 5, and 15; 15 is the greatest.

201. Is there any set of numbers that do not have 1 as a common divisor?

202. Name sets of numbers that have no common factor greater than 1. Such numbers are *prime to each other.*

Ans. 4 and 5; 4 and 9; 4, 8, 25; 8, 9, 10, 12.

203. Are 6, 8, and 10 prime to each other? Why?

204. Are 6, 8, and 9 prime to each other? Why?

In fractions, it is often necessary to find the *greatest common divisor* of two or more numbers. See p. 155.

205. Find the greatest common divisor of 12, 18, and 24.

$$\begin{array}{r|lll} 2 & 12 & 18 & 24 \\ \hline 3 & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 \end{array}$$

2 is a common factor of 12, 18, and 24; 3 is a common factor of the quotients 6, 9, and 12. The quotients, 2, 3, and 4, are prime to each other. Therefore the product of the two divisors, 2×3, or 6, is the greatest common divisor.

Find the greatest common divisor of:

206. 6, 18, 12

207. 10, 15, 20

208. 18, 12, 24

209. 32, 64, 48

210. 60, 120, 30

211. 36, 72, 144

212. 82, 64, 128

213. 14, 28, 42

Multiples

A *multiple* of a number is exactly divisible by that number.

214. Find a multiple of 3. *Ans.* 6; 6 is exactly divisible by 3.

215. Find other multiples of 3.

Ans. 9, 12, 15, 18, 21, and so on.

216. Can you find all the multiples of 3? Why?

217. Name all the multiples of 2 to 100.

218. Name all the multiples of 3 to 99; of 4 to 100; of 5 to 100; of 6 to 96; of 7 to 91.

219. Is there any number which is not a multiple of 1?

A *common multiple* of two or more numbers is exactly divisible by each.

220. Write all the multiples of 2 to 36; all the multiples of 3 to 36.

221. Make a list of all these multiples of 2 that exactly contain 3.

222. Make a list of all these multiples of 3 that exactly contain 2.

223. Make a list of all numbers to 36 that are common multiples of 2 and 3.

224. What is the least common multiple of 2 and 3?

225. Make a list to 72 of all the common multiples of 3 and 4.

226. Make a list to 90 of all the common multiples of 6 and 9.

227. What is the least common multiple of 6 and 9?

228. Make a list of the first twelve common multiples of 8 and 12.

229. What is the least common multiple of 8 and 12?

Least Common Multiple

In fractions, it is often necessary to find the *least common multiple* of two or more numbers. See p. 156.

230. Find the least common multiple of 12 and 18.

$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
$2 \times 2 \times 3 \times 3 = 36$, *L. C. M.*

The least common multiple must contain 12, or $2 \times 2 \times 3$, and we retain these factors. The least common multiple must contain 18, or $2 \times 3 \times 3$. In the factors of 12 we have the 2 and one 3; we retain the other. The least common multiple is $2 \times 2 \times 3 \times 3$, or 36.

231. Find the least common multiple of 2, 3, 4, 6, and 8.

$8 = 2 \times 2 \times 2$
$6 = 2 \times 3$
$4 = 2 \times 2$
$2 \times 2 \times 2 \times 3 = 24$, *L. C. M.*

The least common multiple must contain 8, or $2 \times 2 \times 2$, and we retain these factors.

The least common multiple must contain 6, or 2×3. We have the 2 and retain the 3. The least common multiple must contain 4, or 2×2. We have the two 2's. The least common multiple is $2 \times 2 \times 2 \times 3$, or 24.

Find the L. C. M. of:

232. 8, 10

233. 9, 12

234. 8, 14

235. 8, 12

236. 6, 9, 10

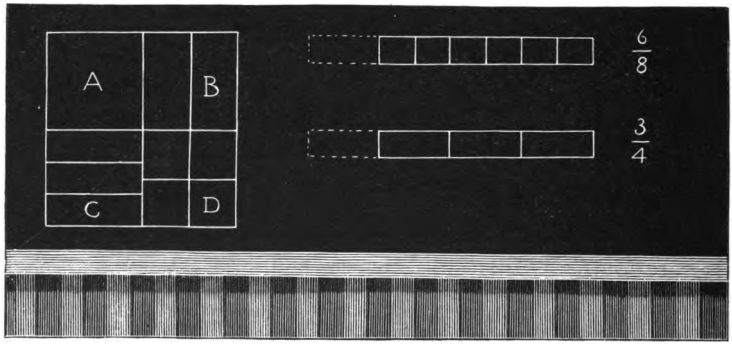
237. 2, 3, 4

238. 8, 9, 12

239. 10, 12, 18

COMMON FRACTIONS

Terms



A *common fraction* is one or more of the equal parts of a unit.

The number showing into how many parts the unit is divided is the *denominator;* it is written below a horizontal line.

The number showing how many parts are taken is the *numerator;* it is written above the horizontal line.

ILLUSTRATION. — 3 quarters of an inch = $\frac{3}{4}$ of an inch. $\frac{3}{4}$ is a common fraction; 3, the numerator; 4, the denominator. It means that an inch has been divided into 4 equal parts, and that 3 of those parts have been taken.

Reduction to Lower Terms

Dividing both numerator and denominator by the same number does not change the value of a fraction.

1. Draw a square, divide into parts, and letter as on the preceding page.

2. What part of the whole is A? B? C? D?

3. Prove that $\frac{4}{16} = \frac{2}{8}$.

Ans. $A = 4D = \frac{4}{16}$; $A = 2B = \frac{2}{8}$. Therefore $\frac{4}{16} = \frac{2}{8}$.

4. Prove that $\frac{2}{8} = \frac{1}{4}$.

Ans. $A = 2B = \frac{2}{8}$; $A = \frac{1}{4}$. Therefore $\frac{2}{8} = \frac{1}{4}$.

5. Draw a line $\frac{6}{8}$ inch long; draw another $\frac{3}{4}$ inch long. Compare the two lines.

6. Prove, by drawing and dividing a rectangle, that $\frac{10}{12} = \frac{5}{6}$.

7. In the same way, prove that $\frac{12}{15} = \frac{4}{5}$.

8. In the same way, prove that $\frac{12}{16} = \frac{3}{4}$.

9. Reduce $\frac{72}{108}$ to lowest terms.

$\frac{72}{108} = \frac{8}{12} = \frac{2}{3}$ We divide both terms of $\frac{72}{108}$ by 9; both terms of $\frac{8}{12}$ by 4.

NOTE. — A fraction is in its *lowest terms* when both numerator and denominator are prime to each other. Thus: $\frac{2}{3}$ is in its lowest terms because 2 and 3 are prime to each other. See p. 151.

Reduce to lowest terms:

10. $\frac{9}{36}$; $\frac{14}{28}$

11. $\frac{27}{36}$; $\frac{18}{36}$

12. $\frac{36}{48}$; $\frac{48}{64}$

13. $\frac{72}{81}$; $\frac{54}{63}$

14. $\frac{27}{81}$; $\frac{96}{98}$

15. $\frac{60}{72}$; $\frac{72}{84}$

16. $\frac{216}{288}$; $\frac{144}{432}$

17. $\frac{196}{210}$; $\frac{256}{272}$

18. $\frac{108}{144}$; $\frac{324}{432}$

Reduction to Higher Terms

Multiplying both numerator and denominator by the same number does not change the value of a fraction.

19. Draw a rectangle as on p. 154, and prove that $\frac{1}{8} = \frac{2}{16}$.

Ans. $B = \frac{1}{8}$; $B = 2D$; $D = \frac{1}{16}$. Therefore $\frac{1}{8} = \frac{2}{16}$.

20. In the same way, prove that $\frac{1}{4} = \frac{3}{12}$.

21. Draw a rectangle; divide it into 12 equal parts; prove that $\frac{2}{3} = \frac{8}{12}$.

22. Change $\frac{2}{3}$ to 12ths.

$\frac{2}{3} = \frac{8}{12}$ To make the denominator 12, we must multiply 3 by 4; multiplying both terms by 4, $\frac{2}{3} = \frac{8}{12}$.

Change:

23. $\frac{5}{6}$ to 12ths; $\frac{5}{9}$ to 45ths; $\frac{3}{14}$ to 42ds.

24. $\frac{3}{4}$ to 12ths; $\frac{5}{8}$ to 72ds; $\frac{2}{15}$ to 60ths.

25. $\frac{3}{8}$ to 64ths; $\frac{5}{7}$ to 49ths; $\frac{7}{19}$ to 57ths.

26. Reduce $\frac{2}{3}$, $\frac{3}{4}$ to equivalent fractions having the *least common denominator*.

$\frac{8}{12}, \frac{9}{12}$ The least common denominator is 12; $12 \div 3 = 4$; we multiply both terms of $\frac{2}{3}$ by 4; $12 \div 4 = 3$; we multiply both terms of $\frac{3}{4}$ by 3.

Reduce to equivalent fractions having L. C. D.:

27. $\frac{3}{8}, \frac{1}{4}$

28. $\frac{2}{3}, \frac{5}{6}$

29. $\frac{1}{2}, \frac{2}{3}$

30. $\frac{3}{4}, \frac{5}{6}$

31. $\frac{3}{4}, \frac{5}{8}$

32. $\frac{2}{9}, \frac{2}{3}$

33. $\frac{5}{6}, \frac{3}{5}, \frac{1}{8}$

34. $\frac{2}{9}, \frac{3}{8}, \frac{1}{3}$

35. $\frac{2}{3}, \frac{3}{7}, \frac{5}{6}$

NOTE. — The *least common denominator* is the least number that will exactly contain the denominators — their *least common multiple*.

Reduction — Whole or Mixed Numbers

A *fraction* is an expression of division. It means that the numerator is to be divided by the denominator.

A *mixed number* is an integer plus a fraction.

36. Reduce $\frac{12}{5}$ to a whole or mixed number.

$\frac{12}{5} = 2\frac{2}{5}$

$\frac{12}{5}$ means $12 \div 5$. Performing the indicated operation, we obtain $2\frac{2}{5}$.

Or, since there are 5 fifths in 1, in 12 fifths there are as many 1's as 5 is contained times in 12, or $2\frac{2}{5}$.

Reduce to a whole or mixed number:

37.	$\frac{8}{3}$; $\frac{91}{8}$	**40.** $\frac{79}{6}$; $\frac{75}{8}$	**43.** $\frac{824}{137}$
38.	$\frac{9}{2}$; $\frac{25}{6}$	**41.** $\frac{81}{9}$; $\frac{69}{4}$	**44.** $\frac{278}{100}$
39.	$\frac{73}{4}$; $\frac{65}{12}$	**42.** $\frac{83}{12}$; $\frac{49}{6}$	**45.** $\frac{338}{168}$

If the integer is reduced to an equivalent fraction having the denominator of the fraction, the two parts may be united. Thus: $4\frac{2}{3} = 4 + \frac{2}{3}$.

46. Reduce $4\frac{2}{3}$ to an improper fraction.

$4\frac{2}{3} = \frac{12}{3} + \frac{2}{3} = \frac{14}{3}$

$4 = 12$ thirds; 12 thirds + 2 thirds = 14 thirds.

Or, since there are 3 thirds in 1, in 4 there are 4 times 3 thirds, or 12 thirds; with 2 thirds, 14 thirds.

NOTE. — An *improper fraction* is a fraction whose numerator equals or exceeds its denominator.

Reduce to a mixed number:

47.	$3\frac{2}{3}$	$6\frac{3}{5}$	$5\frac{1}{6}$	$12\frac{3}{7}$	$12\frac{3}{25}$	$15\frac{2}{13}$
48.	$6\frac{2}{9}$	$5\frac{3}{8}$	$7\frac{4}{9}$	$12\frac{5}{8}$	$19\frac{2}{15}$	$13\frac{5}{14}$
49.	$8\frac{1}{5}$	$9\frac{3}{4}$	$7\frac{8}{9}$	$8\frac{3}{7}$	$30\frac{3}{17}$	$45\frac{2}{19}$

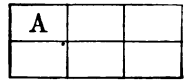
Addition

Before fractions can be added, they must be reduced to equivalent fractions having a common denominator. *See p. 156.*

The least common denominator should always be found.

50. Find the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

1 half = 3 A = 3 sixths
1 third = 2 A = 2 sixths
Sum = 5 A = 5 sixths



$\frac{1}{2}$	3
$\frac{1}{3}$	2
	$\frac{5}{6}$

$\frac{1}{2} = \frac{3}{6}$; $\frac{1}{3} = \frac{2}{6}$; the sum of the 6ths is 5 sixths; we write $\frac{5}{6}$ in fractions' column.

$\frac{1}{2}$	3
$\frac{1}{3}$	2
$\frac{5}{6}$	$\frac{5}{6}$

NOTE. — When the fractions are ready for addition, the work appears as at the left; when finished, as at the right. In practice, it should not be begun in one place and finished in another.

Find the value of:

51. $\frac{2}{9} + \frac{1}{3}$ **53.** $\frac{5}{6} + \frac{3}{4}$ **55.** $\frac{3}{4} + \frac{1}{2} + \frac{5}{9}$

52. $\frac{1}{5} + \frac{2}{3}$ **54.** $\frac{2}{3} + \frac{1}{4}$ **56.** $\frac{5}{8} + \frac{3}{8} + \frac{1}{4}$

57. Find the sum of $8\frac{2}{3}$ and $9\frac{3}{4}$.

$8\frac{2}{3}$	8
$9\frac{3}{4}$	9
$18\frac{5}{12}$	$\frac{17}{12}$

$\frac{2}{3} = \frac{8}{12}$; $\frac{3}{4} = \frac{9}{12}$. The sum of the 12ths is 17 12ths, or 1 unit and 5 12ths; we write 5 12ths in fractions' column, and carry 1 to units' column.

Find the value of:

58. $3\frac{2}{9} + 4\frac{1}{3}$ **61.** $8\frac{2}{3} + 6\frac{1}{4}$ **64.** $3\frac{5}{9} + 8\frac{1}{5} + 4\frac{2}{3}$

59. $5\frac{1}{5} + 8\frac{2}{3}$ **62.** $2\frac{3}{4} + 3\frac{1}{2}$ **65.** $7\frac{4}{5} + 8\frac{5}{6} + 6\frac{1}{2}$

60. $3\frac{5}{6} + 1\frac{3}{4}$ **63.** $1\frac{5}{9} + 4\frac{3}{8}$ **66.** $9\frac{3}{8} + 9\frac{5}{6} + 8\frac{2}{3}$

Problems

67. Charlie gave $\frac{1}{6}$ of his melon to his brother, $\frac{1}{6}$ to his sister, and $\frac{1}{6}$ to his mother. What part of his melon did he give away?

68. A grocer sold $\frac{5}{8}$ of a barrel of sugar to one customer, $\frac{3}{8}$ to another, and $\frac{3}{4}$ to another. How many barrels did he sell altogether?

69. Ned spent $\frac{1}{2}$ of his money for a pair of skates, and $\frac{1}{4}$ of it for a knife. What part of his money did he spend?

70. A lady bought $3\frac{1}{2}$ yards of cloth at one store, and $8\frac{1}{4}$ yards at another. How many yards did she buy altogether?

71. A farmer sold a load of corn for $\$12\frac{3}{4}$, and a ton of hay for $\$5\frac{1}{2}$. How much did he receive for both?

72. John saved $\$\frac{1}{4}$ one week, $\$\frac{3}{4}$ the next, and $\$\frac{1}{2}$ the next. How much did he save in the three weeks?

73. A farmer has three stacks of hay; the first contains $5\frac{1}{2}$ tons, the second $3\frac{3}{4}$ tons, the third $4\frac{5}{8}$ tons. How much hay has he in the three stacks?

74. A fruit dealer bought $23\frac{3}{4}$ dozen bananas, $18\frac{5}{12}$ dozen lemons, and $15\frac{1}{2}$ dozen oranges. How many dozen articles did he buy?

75. A grocer sold $3\frac{7}{12}$ dozen eggs to one man, $5\frac{2}{3}$ dozen to another, and $4\frac{3}{4}$ dozen to another. How many dozen did he sell to the three men?

76. On Monday, a man deposited in the bank, $\$475\frac{3}{4}$; on Tuesday, $\$28\frac{1}{2}$; on Wednesday, $\$19\frac{4}{5}$. How much did he deposit in the bank during the three days?

77. A landowner divided a certain part of a tract of land among his four sons. To the first he gave $\frac{1}{6}$ of the tract; to the second, $\frac{1}{4}$; to the third, $\frac{2}{9}$; and to the fourth, $\frac{5}{12}$. How much of the whole tract was given away?

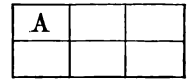
Subtraction

Before fractions can be subtracted, they must be reduced to equivalent fractions having a common denominator. *See p. 156.*

The least common denominator should always be found.

78. Prove that $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

1 half = 3 A = 3 sixths
1 third = 2 A = 2 sixths
Difference = A = 1 sixth



79. From $\frac{1}{2}$ subtract $\frac{1}{3}$.

$\frac{1}{2}$ 3
$\frac{1}{3}$ 2
$\frac{}{\frac{}{6}}$

$\frac{1}{2} = \frac{3}{6}$; $\frac{1}{3} = \frac{2}{6}$; $\frac{2}{6}$ from $\frac{3}{6}$ leaves $\frac{1}{6}$; we write $\frac{1}{6}$ in fractions' column.

$\frac{1}{2}$ 3
$\frac{1}{3}$ 2
$\frac{1}{6}$ $\frac{1}{6}$

NOTE. — When the fractions are ready for subtraction, the work appears as at the left; when finished, as at the right. In practice, it should not be begun in one place and finished in another.

Find the value of:

80. $\frac{3}{4} - \frac{1}{8}$ **82.** $\frac{3}{4} - \frac{3}{8}$ **84.** $\frac{14}{15} - \frac{5}{6}$
81. $\frac{3}{8} - \frac{1}{4}$ **83.** $\frac{2}{3} - \frac{1}{6}$ **85.** $\frac{15}{16} - \frac{3}{5}$

86. From $9\frac{2}{3}$ subtract $3\frac{3}{4}$.

$9\frac{2}{3}$ 12 8
$3\frac{3}{4}$ 9 20
$5\frac{11}{12}$ $\frac{11}{12}$

9 12ths from 8 12ths we cannot take; we take 1 unit from 9 units, leaving 8 units; 1 unit, or 12 12ths with 8 12ths, 20 12ths; 9 12ths from 20 12ths leaves 11 12ths; we write 11 12ths in fractions' column.

3 units from 8 units leaves 5 units; we write 5 in units' column.

Find the value of:

87. $6\frac{2}{3} - 3\frac{1}{2}$ **89.** $3\frac{3}{8} - 1\frac{1}{6}$ **91.** $8\frac{1}{5} - 4\frac{1}{3}$
88. $8\frac{3}{4} - 5\frac{1}{3}$. **90.** $5\frac{1}{4} - 4\frac{3}{5}$ **92** $9\frac{2}{5} - 8\frac{5}{9}$

Problems

93. A man who had $\frac{7}{12}$ of a ton of hay, sold $\frac{2}{3}$ of a ton. What part of a ton had he left?

94. One pail holds $\frac{7}{8}$ of a gallon of milk, and another $\frac{3}{4}$ of a gallon. How much more does the first one hold than the second?

95. A boy earned $\$\frac{3}{4}$, and spent $\$\frac{2}{5}$. How much money had he left?

96. A pail of candy weighed $13\frac{3}{4}$ pounds; the pail, without the candy, weighed $2\frac{7}{8}$ pounds. How many pounds of candy were there in the pail?

97. A farmer sold a cow for $\$43\frac{1}{4}$, and a calf for $\$8\frac{1}{2}$. How much more did he receive for the cow than for the calf?

98. A grain dealer bought corn at $62\frac{2}{3}$ cents a bushel, and sold it at $64\frac{5}{8}$ cents a bushel. What was the gain on each bushel?

99. A telegraph pole, $38\frac{3}{8}$ feet high, after being broken off by a storm, measured $17\frac{1}{2}$ feet. How many feet were broken off?

100. A man spent $\frac{4}{9}$ of his money for a house and lot, $\frac{1}{6}$ for furniture, and $\frac{1}{12}$ for a horse and carriage. What part of his money had he left?

101. The sum of two numbers is $12\frac{5}{16}$; one of the numbers is $8\frac{5}{8}$. What is the other number?

102. From a bolt of muslin containing $25\frac{5}{16}$ yards, $13\frac{5}{8}$ yards were cut at one time, and $4\frac{3}{4}$ yards at another time. How many yards were cut off? How many yards were left in the bolt?

103. From a cask containing $26\frac{3}{4}$ gallons, $6\frac{1}{2}$ gallons were drawn out at one time, $10\frac{1}{4}$ gallons at another time, and $8\frac{5}{8}$ gallons at a third time. How much remained in the cask?

Multiplication

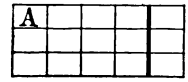
Multiply the numerators for a new numerator and the denominators for a new denominator, canceling when possible.

Mixed numbers should be reduced to improper fractions.

104. Prove that $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$.

$\frac{4}{5} = 12\ A$; $\frac{1}{3}$ of $\frac{4}{5} = 4\ A$

$\frac{2}{3}$ of $\frac{4}{5} = 8\ A = \frac{8}{15}$



105. Multiply $\frac{3}{4}$ by 8.

$\frac{3}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{1} = 6$

Since 8 is a factor of the numerator, and 4 a factor of the denominator, we may divide both terms by 4 (see p. 155), *i.e.*, we may cancel 4.

106. Multiply $\frac{3}{4}$ by $\frac{2}{3}$.

$\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} = \frac{1}{2}$

Find the value of:

107. $\frac{3}{5} \times 15$; $\frac{5}{7} \times 21$

108. $\frac{5}{9} \times 18$; $\frac{3}{8} \times 24$

109. $\frac{5}{6} \times 24$; $\frac{2}{7} \times 14$

110. $\frac{5}{8} \times \frac{16}{35}$; $\frac{3}{7} \times \frac{25}{27}$

111. $\frac{3}{7} \times \frac{14}{33}$; $\frac{4}{7} \times \frac{21}{28}$

112. $\frac{5}{6} \times \frac{18}{55}$; $\frac{5}{8} \times \frac{16}{21}$

113. Multiply 6 by $\frac{2}{3}$.

$\frac{\cancel{6}^{2}}{1} \times \frac{2}{\cancel{3}_{1}} = 4$

114. Multiply $3\frac{2}{3}$ by $2\frac{1}{4}$.

$\frac{\cancel{11}^{\ 3}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{\ 3}}{\cancel{4}_{1}} = \frac{33}{4}$

Find the value of:

115. $30 \times \frac{3}{5}$; $20 \times \frac{4}{5}$

116. $32 \times \frac{7}{8}$; $16 \times \frac{3}{4}$

117. $72 \times \frac{8}{9}$; $80 \times \frac{7}{8}$

118. $2\frac{1}{7} \times 5\frac{1}{4}$; $2\frac{2}{3} \times 2\frac{1}{4}$

119. $5\frac{5}{8} \times 7\frac{1}{9}$; $3\frac{1}{5} \times 1\frac{7}{8}$

120. $9\frac{1}{7} \times 7\frac{7}{9}$; $2\frac{1}{2} \times 3\frac{1}{3}$

Problems

121. At $\$\frac{3}{5}$ a yard, how much will 10 yards of cloth cost?

122. How much will 12 lemons cost at $5\frac{2}{3}¢$ each?

123. A man earns $\$\frac{8}{10}$ in an hour. How much will he earn in 4 hours?

124. A family use $1\frac{1}{4}$ pounds of meat at each meal. How many pounds will they use in 12 meals?

125. How much will 12 tons of hay cost at $\$5\frac{3}{4}$ per ton?

126. When turnips are worth $33\frac{1}{3}¢$ a bushel, how much must I pay for 24 bushels?

127. If the carfare between two places is $\$7\frac{3}{5}$, what will be the fare for 15 passengers?

128. A merchant bought 12 hats at $25\frac{3}{4}¢$ each; 8 neckties at $45\frac{3}{4}¢$ each; and 2 dozen handkerchiefs at $35\frac{3}{8}¢$ each. What was the amount of his bill?

129. How much will $\frac{3}{4}$ of a ton of hay cost at $\$7\frac{3}{4}$ a ton?

130. A lady bought $3\frac{1}{2}$ yards of velvet at $\$1\frac{3}{4}$ a yard; $6\frac{3}{4}$ yards of silk at $\$1\frac{1}{2}$ a yard. What was the amount of her bill?

131. If a man travels on a wheel $\frac{2}{5}$ of 95 miles in one day, how far will he travel in $6\frac{1}{19}$ days at the same rate?

132. A and B start from the same place and walk in opposite directions, one at the rate of $7\frac{1}{3}$ miles a day and the other at the rate of $6\frac{4}{5}$ miles a day. How far apart will the men be at the end of the second day?

133. From a box containing 6 dozen eggs, a grocer sells to one customer $\frac{1}{3}$ of the entire number, and to a second $\frac{3}{4}$ as many eggs as he sold to the first customer. How many eggs does he sell to each customer? how many dozen to the second?

134. If I sell cherries at $72¢$ a bushel, how much will I receive for $\frac{1}{6}$ of a bushel? $\frac{3}{8}$ of a bushel? $\frac{5}{9}$ of a bushel?

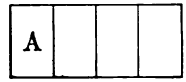
Division — Common Method

Invert the divisor and proceed as in multiplication.

135. Prove that $1 \div \frac{3}{4} = \frac{4}{3}$, or that dividing by a fraction is multiplying by its reciprocal.

$$1 = 4A; \quad \frac{3}{4} = 3A$$

$$1 \div \frac{3}{4} = 4A \div 3A = \frac{4}{3}$$



136. Divide $\frac{1}{2}$ by $\frac{3}{4}$.

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{3} = \frac{2}{3}$$

Since dividing by $\frac{3}{4}$ is multiplying by $\frac{4}{3}$, we invert the divisor and proceed as in multiplication.

137. Divide $\frac{7}{9}$ by 14.

$$\frac{\cancel{7}}{9} \times \frac{1}{\cancel{14}_{2}} = \frac{1}{18}$$

138. Divide $18\frac{3}{4}$ by $5\frac{5}{6}$.

$$\frac{\overset{15}{\cancel{75}}}{\underset{2}{\cancel{4}}} \times \frac{3}{\underset{7}{\cancel{35}}} = \frac{45}{14} = 3\frac{3}{14}$$

Find the value of:

139. $\frac{2}{3} \div \frac{2}{5}$	**142.** $\frac{7}{8} \div 7$	**145.** $3\frac{2}{3} \div 5\frac{1}{2}$
140. $\frac{5}{6} \div 5$	**143.** $8 \div \frac{4}{7}$	**146.** $3\frac{1}{3} \div 7\frac{1}{7}$
141. $\frac{2}{3} \div \frac{5}{6}$	**144.** $4 \div \frac{4}{9}$	**147.** $8\frac{2}{5} \div 1\frac{1}{5}$

Find the value of:

148. $\frac{3}{8} \div 6$	**151.** $\frac{3}{7} \div 6$	**154.** $8\frac{1}{3} \div 2\frac{7}{9}$
149. $\frac{3}{4} \div \frac{1}{2}$	**152.** $\frac{5}{8} \div 5$	**155.** $3\frac{5}{6} \div 2\frac{1}{2}$
150. $\frac{5}{6} \div \frac{5}{6}$	**153.** $8 \div \frac{8}{9}$	**156.** $6\frac{3}{4} \div 6\frac{1}{2}$

Problems

157. If $\frac{3}{4}$ of a yard of cloth costs $12¢$, how much will one yard cost?

Analysis: Since $\frac{3}{4}$ of a yard costs $12¢$, $\frac{1}{4}$ of a yard will cost $\frac{1}{3}$ of $12¢$, or $4¢$; $\frac{4}{4}$, or 1 yard, will cost 4 times $4¢$, or $16¢$.

158. If $\frac{5}{8}$ of a yard of ribbon costs $15¢$, what will be the cost of one yard?

159. Find the cost of 1 bushel of potatoes, if $\frac{3}{4}$ of one bushel costs $.48.

160. What is the cost of 1 bushel of apples, if $\frac{2}{3}$ of a bushel costs $.36?

161. If $\frac{3}{4}$ of a pound of candy is given to each child, how many children will 6 pounds supply?

162. If $\frac{5}{8}$ of a bushel of nuts is placed in each bag, how many bags will be required for 10 bushels?

163. If 3 dozen bananas can be bought for $\$1\frac{8}{25}$, what is the price of each dozen? What is the price in cents of one banana?

164. If 1 man can build a fence in $5\frac{5}{11}$ days, in what time can 12 men build it? In what time can 3 men do the same work?

165. A field containing $27\frac{5}{9}$ square rods was divided into lots, each of $3\frac{4}{9}$ square rods. How many lots were there?

166. A farmer had $56\frac{7}{18}$ bushels of wheat, which he put into 5 bins of equal size. How many bushels did he put into each bin?

167. Mary had 2 oranges, her brother gave her $\frac{1}{2}$ of his orange, then she divided all that she had equally among 3 playmates. How much did each of her playmates receive?

168. John had $2\frac{5}{8}$ quarts of cherries, which he shared equally with his 3 playmates. How much did each receive?

Miscellaneous

Find the value of:

169. $\frac{2}{3} + \frac{3}{4}$

170. $\frac{3}{8} + \frac{5}{6}$

171. $\frac{2}{5} + \frac{3}{7}$

172. $\frac{5}{9} + \frac{2}{3}$

173. $2\frac{5}{6} + 1\frac{7}{9}$

174. $7\frac{3}{5} + 1\frac{2}{3}$

175. $5\frac{4}{9} + 1\frac{5}{8}$

176. $2\frac{3}{5} + 1\frac{1}{2}$

Find the value of:

177. $\frac{5}{6} - \frac{2}{3}$

178. $\frac{8}{9} - \frac{5}{6}$

179. $\frac{3}{4} - \frac{5}{9}$

180. $\frac{6}{7} - \frac{3}{8}$

181. $5\frac{1}{3} - 2\frac{1}{2}$

182. $6\frac{2}{3} - 1\frac{3}{4}$

183. $8\frac{1}{2} - 3\frac{5}{6}$

184. $7\frac{3}{8} - 6\frac{3}{4}$

Find the value of:

185. $\frac{5}{8} \times \frac{4}{35}$

186. $\frac{9}{14} \times \frac{7}{18}$

187. $\frac{3}{5} \times \frac{10}{21}$

188. $\frac{5}{7} \times \frac{21}{35}$

189. $6 \times 12\frac{2}{3}$

190. $7\frac{1}{9} \times 12$

191. $8\frac{4}{5} \times 5\frac{5}{11}$

192. $8\frac{2}{3} \times 1\frac{8}{13}$

Find the value of:

193. $\frac{5}{9} \div \frac{1}{18}$

194. $\frac{7}{8} \div 14$

195. $8 \div 4\frac{2}{5}$

196. $6 \div 8\frac{4}{7}$

197. $1\frac{7}{8} \div 1\frac{1}{4}$

198. $6\frac{3}{7} \div 3\frac{1}{9}$

199. $2\frac{2}{7} \div 9\frac{1}{3}$

200. $3\frac{4}{7} \div 3\frac{1}{8}$

Problems

201. 15 is $\frac{3}{4}$ of some number. What is $\frac{1}{4}$ of the number?

Ans. $\frac{1}{3}$ of 15, or 5.

202. 15 is $\frac{3}{4}$ of what number?

Ans. Since $\frac{3}{4}$ of the number is 15, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 15, or 5; $\frac{4}{4}$, or the number, is 4 times 5, or 20.

203. 15 is $\frac{5}{8}$ of what number? 20 is $\frac{5}{8}$ of what number? 18 is $\frac{3}{7}$ of what number?

204. 12 is $\frac{3}{7}$ of what number? 25 is $\frac{5}{9}$ of what number? 10 is $\frac{5}{9}$ of what number? 18 is $\frac{6}{7}$ of what number?

205. John is 12 years old, or $\frac{2}{3}$ as old as James. How old is James? Prove.

206. I have 6 apples; this number is $\frac{2}{3}$ of the number which Alice has. How many apples has Alice? Prove.

207. John and James together have 21¢; James has $\frac{3}{4}$ as much money as John. How much has each?

Ans. They both have $\frac{7}{4}$ × John's money. Since $\frac{7}{4}$ × John's money is 21¢, etc.

208. The difference between the ages of John and Henry is 12 years, and John is $\frac{1}{3}$ as old as Henry. How old is each?

SUGGESTION. — The difference between their ages is $\frac{2}{3}$ of Henry's age.

209. A and B together own a boat worth $500. A's share is $\frac{2}{3}$ of B's. What is the value of A's share? Prove.

210. C owns $\frac{2}{3}$ of a ship, and D owns $\frac{1}{4}$. What part do they both own?

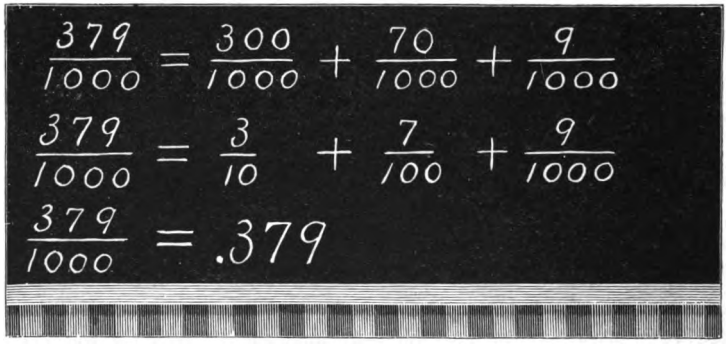
211. If the value of the ship is $21,000, by how much does the value of C's part exceed the value of D's part?

212. E owns $\frac{5}{12}$ of a building, and F owns $\frac{1}{2}$; the value of both their shares is $12,100. What is the value of F's share? Prove.

213. John is worth $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $2400. How much is John worth?

DECIMALS

Terms



A *decimal fraction* is a fraction whose denominator is 10, 100, 1000, or 1 with some number of ciphers.

A decimal fraction may be resolved into a series of fractions whose denominators are 10, 100, 1000, . . .

1. Resolve $\frac{379}{1000}$ into a series of fractions whose denominators are 10, 100, and 1000. *Ans.* $\frac{3}{10} + \frac{7}{100} + \frac{9}{1000}$.

2. Read $\frac{379}{1000}$ as a series of fractions whose denominators are 10, 100, and 1000. *Ans.* 3 tenths 7 hundredths 9 thousandths.

3. Read $\frac{3759}{10000}$ as a series of fractions whose denominators are 10, 100, 1000, and 10000.

Ans. 3 tenths 7 hundredths 5 thousandths 9 ten-thousandths.

4. Name the decimal orders.

Ans. Tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, and so on.

Notation and Numeration

To simplify the writing of a series of fractions whose denominators are 10, 100, 1000, and so on, it is agreed that a decimal point (.) shall be placed before "tenths," and that the other figures shall express in order, hundredths, thousandths, ten-thousandths, etc.

5. Write "3 tenths 7 hundredths 9 thousandths" in this way. *Ans.* .379

6. Write $\frac{3}{10} + \frac{7}{100} + \frac{9}{1000}$ in this way. *Ans.* .379

7. Write $\frac{379}{1000}$ in this way. *Ans.* .379

8. Write "3 hundredths" as a decimal.

Ans. .03; 3 hundredths = 0 tenths 3 hundredths, or .03

---

9. Gather $\frac{3}{10} + \frac{7}{100} + \frac{9}{1000}$ into a single fraction.

Ans. $\frac{3}{10} + \frac{7}{100} + \frac{9}{1000} = \frac{379}{1000} = .379$

10. Gather $\frac{3}{10} + \frac{7}{100} + \frac{5}{1000} + \frac{9}{10000}$ into a single fraction.

Ans. $\frac{3}{10} + \frac{7}{100} + \frac{5}{1000} + \frac{9}{10000} = \frac{3759}{10000} = .3759$

11. Observe, in the above, that the denominator of the single fraction is the same as the denominator of the last fraction of the series.

12. What is the denominator of .3798?

Ans. Ten thousand; we numerate from "tenths" to the last figure; tenths, hundredths, thousandths, ten-thousandths.

NOTE. — The denominator of $\frac{3798}{10000}$ is ten thousand; not ten-thousandths.

13. What is the denominator of .13? .03? .285? .002?

14. What is the denominator of .2345? .0006? .23458? .5? .50? .500? .5000?

Numeration

Read a decimal as a common fraction; first its numerator, then its denominator in the ordinal form.

Read the numerator, the denominator, the fraction:

15. .05	**17.** .0238	**19.** .3508
16. .005	**18.** .0028	**20.** .2780

Ex. **15.** The numerator is 5; the denominator, 100; the fraction is read, 5 hundredths.

Ex. **17.** The numerator is 238; the denominator 10000; the fraction is read, 238 ten-thousandths.

Read:

21. .25	**27.** .9252	**33.** .8403
22. .890	**28.** .2178	**34.** .1672
23. .375	**29.** .00004	**35.** .20043
24. .0062	**30.** .87892	**36.** .20821
25. .8252	**31.** .902056	**37.** .314068
26. .0002	**32.** .9260	**38.** .0415

Read:

39. 8.5	**44.** 19.40	**49.** 17.24
40. 6.25	**45.** 8.25	**50.** 3.65
41. 5.3	**46.** 978.36	**51.** 698.33
42. 73.89	**47.** 52.14	**52.** 25.81
43. 78.00	**48.** 83.7	**53.** 781.43

Note. — In reading mixed decimals, the word "and" should be used at the decimal point, but nowhere else. Without this understanding, *four hundred and six thousandths* might be written either 400.006, or .406.

Notation

Write the numerator as in common fractions, and place the decimal point so as to make the name of the last order the name of the denominator.

54. Write 23 ten-thousandths.

.0023

We write the numerator as in common fractions, 23; looking at 3, we think *ten-thousandths* because this is the name of the denominator; at 2, *thousandths;* supplying 0 (it now appears 023), we think *hundredths;* supplying 0 (it now appears .0023), we think *tenths;* writing the decimal point, we have .0023.

Write decimally:

55. 5 tenths.
56. 5 hundredths.
57. 5 thousandths.
58. 5 ten-thousandths.
59. 625 ten-thousandths.
60. 92 thousandths.
61. 825 millionths.
62. 6205 ten-thousandths.
63. 10 hundredths.
64. 10 thousandths.
65. 10 ten-thousandths.
66. 10 millionths.
67. 15 thousandths.
68. 63 hundredths.
69. 1 thousandth.
70. 615 thousandths.

Write decimally:

71. Nine hundred twenty, and sixteen hundredths.
72. Seven hundred eighteen, and four thousandths.
73. Ten thousand eight hundred ten, and five thousandths.
74. Nine hundred nineteen thousand seventeen, and six ten-thousandths.
75. Twenty-five thousand twenty-five, and seventy-five hundred-thousandths.

Addition and Subtraction

Write units of the same order in the same column.

76. Add: 2.06, 35.6, 28.895.

$$\begin{array}{r} 2.06 \\ 35.6 \\ 28.895 \\ \hline 66.555 \end{array}$$

The explanation is the same as on p. 110. The sum of the thousandths, etc.

Add:

77. 625.81, 8.808, 53.675, 91.091, 70.86, 8920.6

78. 9706.005, 8180.0028, 21.21, 893.0056

79. 87.0096, 89572.17, 7654.321, 91.3124

80. .78318, 923.42, 859.021, 19.06, 7.17964

81. \$219.08, \$8.96, \$3125.215, \$38.75, \$391.0046

82. \$3742.25, \$6.75, \$128.39, \$76.125, \$371.0602

—•—

83. From 43.2 subtract 9.368.

$$\begin{array}{r} 43.200 \\ 9.368 \\ \hline 33.832 \end{array}$$

43.2 = 43.200, because there are *no* hundredths and *no* thousandths.

The explanation is the same as on p. 122. 8 thousandths from 0 thousandths we cannot take, etc.

Find the value of:

84. 299.28 − 18.65

85. 79.3476 − 9.032

86. 493.83 − 29.835

87. 914.115 − 425.08

88. 436.125 − 4.3395

89. 8325.04 − 634.166

90. 8276.65 − 401.625

91. 4.28 − 2.335

92. 4728 − 15.8

93. 563.08 − 308.461

94. 9001.06 − 1154.187

95. 79.03 − 9.7365

Multiplication

Multiply as in integers, and point off as many decimal places in the product as there are decimal places in both multiplicand and multiplier.

96. Multiply .23 by .5, and explain.

$$\begin{array}{r} .23 \\ .5 \\ \hline .115 \end{array}$$

This is an example in multiplication of fractions. To multiply fractions, we multiply the numerators for a new numerator, and the denominators for a new denominator. The numerators are 23 and 5; we multiply them as in integers; their product is 115.

The denominators are 100 and 10; their product is "1 with as many ciphers" as there are in both. Therefore we point off as many decimal places as there are in both multiplicand and multiplier.

Multiply:

97. $375 \times .5$

98. $3.75 \times .5$

99. $37.5 \times .5$

100. $.375 \times .05$

101. $3.75 \times .05$

102. $375 \times .005$

103. 3.75×5

104. $3.75 \times .005$

105. $.426 \times 3.05$

106. 97.3×4.004

107. $85.627 \times .4632$

108. $92.675 \times .0025$

109. 435.64×4.06

110. 785.6×47.82

111. $.9235 \times .435$

112. $80.027 \times .35$

Multiply:

113. Six, and three thousand two hundred eighty-one thousandths; by sixteen hundred, and sixteen hundredths.

114. Twelve hundred; by thirty-three hundredths.

115. Five hundred seven, and four hundred seventy-six ten-thousandths; by sixty-five, and eight thousandths.

Division

Divide as in integers, and point off as many decimal places in the quotient as those in the dividend exceed those in the divisor.

116. Divide .115 by .23

$$.23\overline{).115(.5}$$
$$\underline{115}$$

This is an example in division of fractions. To divide fractions, we divide the numerators for a new numerator and the denominators for a new denominator.

The numerators are 115 and 23; we divide them as in integers; their quotient is 5.

The denominators are 1000 and 100; their quotient is "1 with as many ciphers" as those in the dividend exceed those in the divisor. Therefore we point off as many decimal places in the quotient as those in the dividend exceed those in the divisor.

NOTE. — We use short division when the numerator of the divisor is less than 13. Thus, in examples 117 to 120, it is best to use short division.

Find the value of:

117. $225.72 \div 9$

118. $225.72 \div .9$

119. $225.72 \div .09$

120. $22.572 \div .009$

121. $45.664 \div 57.08$

122. $22.526 \div 3.218$

123. $126.42 \div 21.07$

124. $.36323 \div .5189$

Find the value of:

125. $129.591 \div .63$

126. $69.0624 \div 3.6$

127. $45.0144 \div 36$

128. $2804.40 \div 7.2$

129. $26.740 \div .764$

130. $25.032 \div 59.6$

131. $.12816 \div 356$

132. $20.703 \div 3.09$

Division

133. Divide .64 by .032

$$\begin{array}{l} .032).640(2. \\ \underline{640} \end{array}$$

If the number of decimal places in the divisor exceeds those in the dividend, we annex ciphers to the dividend until the number of places equals the number in the divisor.

In .64 ÷ .032, we can divide the numerators for a new numerator, but cannot divide the denominators for a new denominator, because 100 does not contain 1000. We must therefore reduce to equivalent fractions having the least common denominator. (See p. 156.) Annexing ciphers to the dividend does this. Thus, .64 ÷ .032 = .640 ÷ .032.

Find the value of:

134. 36.9 ÷ .003

135. 72.54 ÷ .018

136. 8.551 ÷ .0017

137. 8127 ÷ .27

138. 97.28 ÷ .256

139. 203.5 ÷ 17.2

140. Divide 486 by 216.

$$\begin{array}{l} 216)486.00(2.25 \\ \underline{432} \\ 540 \\ \underline{432} \\ 1080 \\ \underline{1080} \end{array}$$

We write a decimal point after the whole number, 486, and are careful to place a cipher in the dividend every time we annex a cipher below.

Thus, 54 is not divided by 216; we annex a cipher to 54 and place another in the dividend.

Find the value of:

141. 38642 ÷ 124

142. 73928 ÷ 268

143. 86935 ÷ 325

144. 450 ÷ 64

145. 16 ÷ 128

146. 38 ÷ 152

Division — Common Fractions to Decimals

147. Divide 37.5 by 1000.

$37.5 \div 1000 = .0375.$ Moving the decimal point one place to the left divides by 10; two places, by 100; etc.

148. Divide 78 by 2000.

$$2)\underline{.078}$$
$$.039$$

First, we divide by 1000, and then by 2.

149. Divide 146.4 by 30000; 14.64 by 3000.

150. Divide .387 by 900; 3.87 by 9000; 3.87 by 90.

151. Reduce $\frac{5}{8}$ to a decimal.

$$8)\underline{5.000}$$
$$.625$$

$\frac{5}{8}$ means $5 \div 8$; performing the indicated operation, we obtain .625.

Reduce to a decimal:

152. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$

153. $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$

154. $\frac{5}{8}$; $\frac{7}{8}$; $\frac{1}{16}$

155. $\frac{3}{16}$; $\frac{5}{16}$; $\frac{7}{16}$

156. Reduce $\frac{2}{7}$ to a decimal.

I. Result exact.

$$7)\underline{2.000}$$
$$.285\tfrac{5}{7}$$

If the exact result is required, it is customary to carry out the division a few places, and to write the remainder as in I.

II. Result approximate.

$$7)\underline{2.000}$$
$$.285+$$

If an approximation is desired, in place of the remainder, we write " + " as in II.

157. Reduce to a decimal: $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{1}{9}$; $\frac{2}{9}$.

Decimals to Common Fractions

Write the decimal as a common fraction and reduce to lowest terms.

Reduce to common fractions:

158. .16; .25
159. .64; .625
160. .75; .375
161. .075; .128
162. .032; .175
163. .010; .005

Ex. **158.** $.16 = \frac{16}{100} = \frac{4}{25}$.

Add:

164. 68, 98.4, 36.5, 73.2, 38.25, 70.05
165. 120.2, 136.002, 385, 386.95, 278.0005
166. 27.07, 30.03, 7865.0005, 32.08, 101.0101
167. 36.3606, 38.5, 3.92, 392, .392, 39.02

Subtract:

168. 26.02 from 82.01
169. 38.065 from 100
170. 1.0004 from 1.032
171. 73.846 from 74
172. 2.0326 from 3
173. 26.87 from 268.7

Multiply:

174. 36.154 by 3.26
175. 326.01 by 3.01
176. 29.806 by 1.03
177. 302.06 by 1.98
178. 40.065 by 12.5
179. 27.036 by 37.85

Divide:

180. .072 by .16
181. .972 by 4.86
182. 145.11 by .215
183. .0068 by .034
184. 10.506 by 10.2
185. .0037 by 1.25

Bills

When a person buys or sells, a statement or bill is usually prepared. *Dr.* means, "He owes me"; *Cr.*, "I owe him."

186. Jan. 23, 1899, Mrs. A. R. Brown bought of George W. Walters & Co., of Springfield, Mass.: 30 lb. granulated sugar @ 8¢; 4 lb. of codfish @ $6\frac{1}{4}$¢; 2 sacks of flour @ $1.65; 6 bars of soap @ $4\frac{1}{6}$¢; 3 lb. of coffee @ 35¢. She gave in exchange: 12 lb. of butter @ 22¢; 24 doz. eggs @ $12\frac{1}{2}$¢. Make out the bill.

Springfield, Mass., Jan. 23, 1899.

Mrs. A. R. Brown,

To George W. Walters & Co. Dr.

1899			
Jan. 23	To 30 lb. gran. sugar @ 8¢,	$2	40
	" 4 lb. codfish @ $6\frac{1}{4}$¢,		25
	" 2 sacks flour @ $1.65,	3	30
	" 6 bars of soap @ $4\frac{1}{6}$¢,		25
	" 3 lb. coffee @ 35¢,	1	05
		7	25
	Cr.		
Jan. 23	By 12 lb. butter @ 22¢,	$2	64
	" 24 doz. eggs @ $12\frac{1}{2}$¢,	3	00
		5	64
	To Balance,	$1	61

Bills

Make out the following bills:

187. Mar. 3, 1898, Henry Lyman bought of William Clarke, Emporia, Kan.: 12 chairs @ $2.50; 3 rockers @ $5.75; 2 mattresses @ $4.50. Apr. 4, he bought: 3 washstands @ $6.25; 4 tables @ $2.75.

188. Jan. 2, 1898, Mrs. L. L. Corbett bought of Jones & Stone, Galesburg, Ill.: 16 bu. of potatoes @ 87½¢; 3 bu. turnips @ 40¢; 26 bbl. apples @ $3.75; 7½ lb. butter @ 16¢; 3 lb. cheese @ 18¢; 25 lb. crackers at 8¢; 5 sacks of flour @ $1.65; 3 bottles vanilla @ 25¢; 6 hams, 65¾ lb. @ 12½¢; 7 doz. eggs @ 12¢; 13 lb. mackerel @ 12¢; 3 doz. oranges @ 35¢.

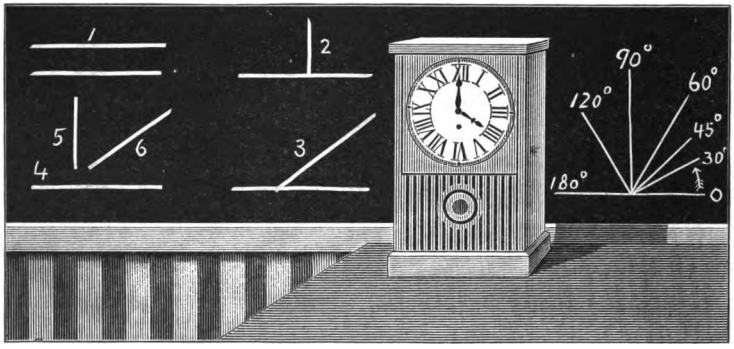
189. Feb. 1, 1898, E. H. Chapin of Hartford, Conn., worked 6½ hr. @ 40¢ for James H. Smith. Feb. 3, he worked 3¾ hr. @ 40¢; Feb. 4, 2¼ hr. @ 40¢, and furnished 24 ft. boards @ 4¢; Feb. 5, 5¾ hr. @ 40¢, and furnished 128 ft. boards @ 4¢; Feb. 6, 8 hr. @ 40¢.

190. Mar. 2, 1898, James Lakin bought of William Matthews, Meriden, Conn.: 3¼ lb. porter house steak @ 25¢; 3 lb. sausage @ 15¢. Mar. 3, he bought: 11¼ lb. of ham @ 16¢; 16 lb. roast beef @ 22½¢; 14 lb. of pork @ 14¢. Mar. 6, he bought: 12 lb. veal @ 26¢; 8¾ lb. mutton chops @ 23¢. Mar. 7, he paid $5 on account.

191. May 14, 1898, Mrs. H. A. Sheppard, San Francisco, Cal., bought: 2 bbl. flour @ $6.25; 18 cans of peaches @ 12½¢; 24 cans tomatoes @ 8¢; 96 lb. granulated sugar @ 12½¢; 2 lb. of tea @ 55¢; 14 lb. of brown sugar @ 8¢. She gave in exchange: 1 bbl. vinegar, 40 gal., @ 18¢; 10 bu. potatoes @ 45¢.

MEASUREMENTS

Lines and Angles



1. Draw two straight lines that meet. They form *angles*.

2. Draw two straight lines that will not meet however far they are extended. They are *parallel*, (1).

3. Draw two straight lines that meet so as to make the angles equal. Each is a *right angle;* the lines are *perpendicular* to each other, (2).

4. Draw two straight lines that meet so as to make the angles unequal. The larger is an *obtuse angle;* the smaller, an *acute angle*, (3).

5. Draw a line parallel to the horizon; it is a *horizontal line*, (4).

6. Draw a line perpendicular to the horizon; it is a *vertical line*, (5).

7. Draw a line neither parallel nor perpendicular to the horizon; it is an *oblique line*, (6).

Lines and Angles

Lines have length. Amount of length is expressed by the measures in the following table, called *Long Measure.* See p. 26.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
5½ yards	= 1 rod (rd.).
320 rods	= 1 mile (mi.).

8. How many yards in 4 rd.? How many feet in 4 rd.?

9. How many rods in 33 ft.? miles in 640 rd.?

10. How many inches in 2 rd. 3 yd. 2 ft. 6 in.? feet in 1 mi.?

Angles have different degrees of opening. Amount of opening is expressed by the measures in the following table, called *Circular Measure.*

60 seconds (″)	= 1 minute (′).
60 minutes	= 1 degree (°).
360 degrees	= 1 circumference (C.).

11. Draw a clock face with the hands reading 12 o'clock. What angle do the hands make? *Ans.* No angle.

12. Draw a clock face with the hands reading 1 o'clock.

13. From 12 o'clock to 1 o'clock the minute hand makes a complete revolution and passes through 360 degrees (360°). How many degrees of opening are there between the hands at 1 o'clock? *Ans.* 30°, $\frac{1}{12}$ of 360°.

14. How large is the angle at 2 o'clock? at 3? at 6?

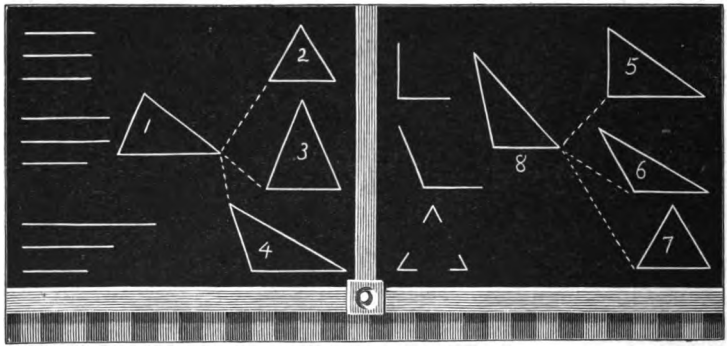
15. At what exact hours are the hands at right angles?

16. How many: seconds in 3′ 20″? minutes in 4° 40′?

17. How many: minutes in 480″? degrees in 540′?

18. How many: seconds in 2° 28′ 36″? in 3° 3′ 3″? in 5° 5″?

Surfaces



19. Inclose a surface with the least possible number of straight lines. You have drawn a *triangle*, (1).

20. Draw sets of three lines. What are their relative lengths? *Ans.* All three are equal, two are equal, or no two are equal.

21. Draw a triangle which has all three of its sides equal and name it *equilateral triangle*, (2).

22. Draw a triangle which has only two of its sides equal and name it *isosceles triangle*, (3).

23. Draw a triangle which has no two of its sides equal and name it *scalene triangle*, (4).

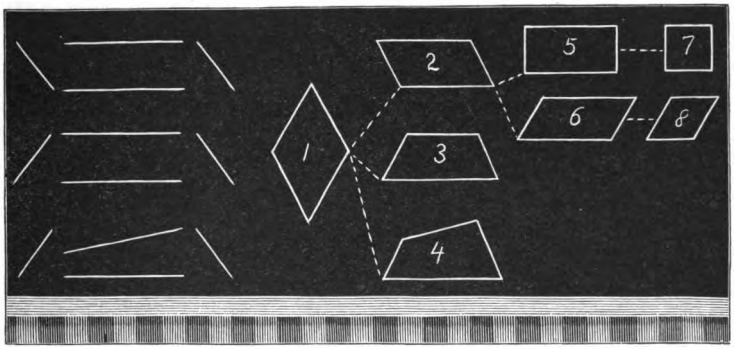
24. How many right angles may a triangle have? how many obtuse angles? how many acute angles? Draw a triangle with one right angle and name it *right-angled triangle*, (5).

25. Draw a triangle with one obtuse angle and name it *obtuse-angled triangle*, (6).

26. Draw a triangle with three acute angles and name it *acute-angled triangle*, (7).

27. Write a cross on that side of each of your triangles on which it seems to rest. Name that side *base*, (8).

Surfaces



28. Inclose a surface with four straight lines. You have a *quadrilateral*, (1).

29. Draw sets of two pairs of opposite lines. What are their relations? *Ans.* Both pairs are parallel, one pair is parallel, or neither pair is parallel.

30. Draw a quadrilateral which has both pairs of its opposite sides parallel, and name it *parallelogram*, (2).

31. Draw a quadrilateral which has only one pair of its opposite sides parallel, and name it *trapezoid*, (3).

32. Draw a quadrilateral which has neither pair of its opposite sides parallel, and name it *trapezium*, (4).

33. Show that the angles of a parallelogram must be right angles or not right angles.

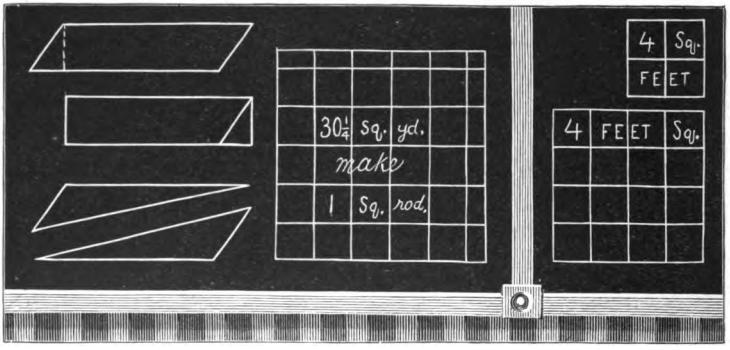
34. Draw a parallelogram which has its angles right angles, — a *rectangle*, (5).

35. Draw a parallelogram which has its angles not right angles, — a *rhomboid*, (6).

36. Draw a rectangle with equal sides, — a *square*, (7).

37. Draw a rhomboid with equal sides, — a *rhombus*, (8).

Surfaces



Surfaces have area, *i.e.*, they are made up of square units. Amount of area is expressed by the measures in the following table, called *Square Measure.* See p. 67.

144 square inches (sq. in.)	**= 1 square foot (sq. ft.).**
9 square feet	**= 1 square yard (sq. yd.).**
30¼ square yards	**= 1 square rod (sq. rd.).**
160 square rods	**= 1 acre (A.).**
640 acres	**= 1 square mile (sq. mi.).**

38. Prove that 144 sq. in. = 1 sq. ft. See p. 68.

39. Prove that 9 sq. ft. = 1 sq. yd. See p. 68.

40. Draw a square 5 in. by 5 in. and divide it into square inches. How many square inches?

41. Prolong the length ½ inch and the width ½ inch; prolong the lines which divide into squares. How many square inches in the 10 rectangles thus formed? What part of a square inch is the small square? How many square inches in the whole? What is $5\frac{1}{2} \times 5\frac{1}{2}$?

42. Let each unit of length represent a yard. How many square yards make 1 sq. rd.?

Surfaces

43. 320 rd. make 1 mi. How many sq. rd. make 1 sq. mi.?

44. Since 160 sq. rd. make 1 A., how many acres make 1 sq. mi.?

45. How many square yards in 18 sq. ft.? 27 sq. ft.?

46. How many square rods in a field containing 3 A.?

47. What is the difference between 4 feet square and 4 square feet? Illustrate by drawing.

48. On paper draw a parallelogram and its altitude; cut out the parallelogram; cut off the triangle.

NOTE. — The dotted line in the illustration is the altitude of the parallelogram.

49. Place the two parts together so as to make a rectangle; so as to make a parallelogram. What is the relation of the rectangle to the parallelogram as to size?

50. What is the area of a rectangle? of a parallelogram?

Ans. The product of the base by the altitude.

51. Cut out a parallelogram, and cut the parallelogram along its diagonal. Compare the triangles as to size.

52. What is the area of a triangle?

Ans. One half the product of its base by its altitude.

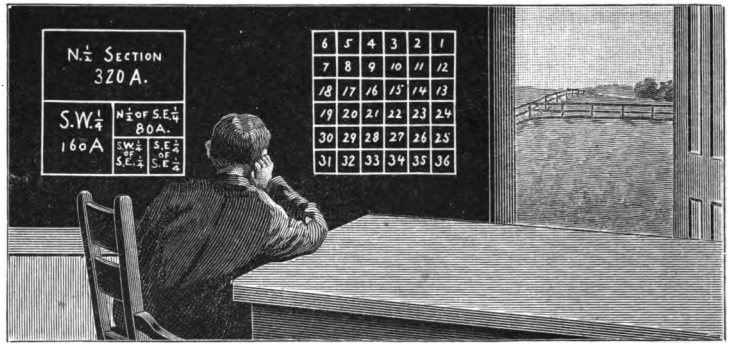
53. Find the area of a parallelogram whose base is 8 in. and altitude 5 in. Draw the figure.

54. Find the area of an oblique-angled triangle whose base is 10 ft. and altitude 6 ft. Draw the figure.

55. Find the area of a right-angled triangle whose base is 18 rd. and altitude 9 rd. Draw the figure.

56. Find the area of an acute-angled triangle whose base is 20 mi. and altitude 12 mi.

Public Lands



The land in western states has been divided into squares 6 mi. each way, called *townships;* and each township has been divided into squares 1 mi. each way, called *sections.*

57. Draw a square to represent a township. How long is a township? how wide?

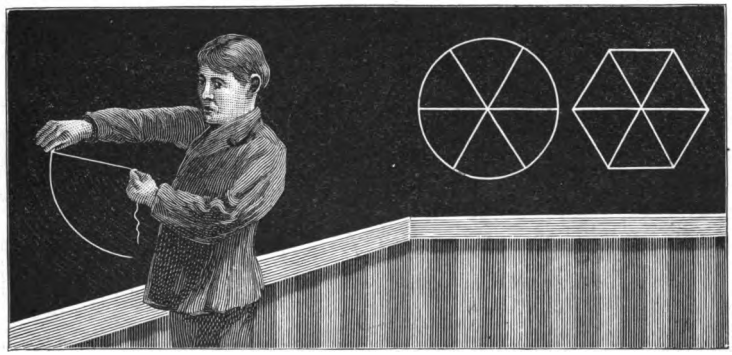
58. Divide the township into squares, each side of which shall represent a mile. How many small squares in the township?

59. Draw a line to represent 1 mile. Upon this line construct a square and divide it into two equal parts by an east and west line. Upon the south part write the name. How many rods long is a half section? How many rods wide? How many acres in a half section?

60. Divide the N. $\frac{1}{2}$ section into 2 equal squares by a north and south line. Upon the N.E. square, write the name. What are the dimensions of a quarter section in rods? How many acres in a quarter section?

61. Divide the N.W. $\frac{1}{4}$ section into 2 equal parts by a north and south line. Upon each part write its name.

Circle



The circumference of a circle is 2 *times the radius times* 3.1416.

The area of a circle is $\frac{1}{2}$ *the product of the circumference by the radius.*

62. Tie a string to a pencil; an inch from the pencil fasten the string to a pin; draw a circle.

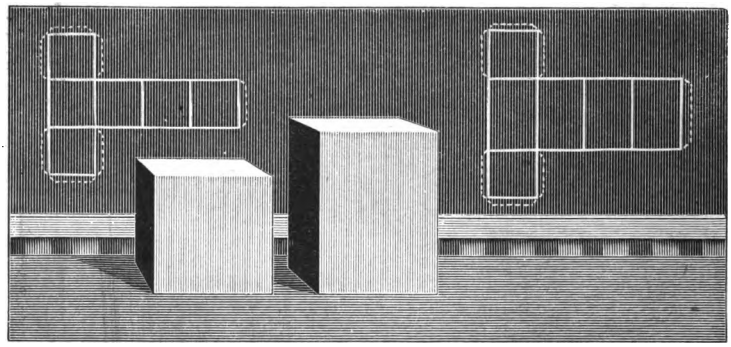
63. Cut a circle out of pasteboard. Measure its circumference (length of its bounding line) with a string and rule; measure its diameter; divide the circumference by the diameter. How does your result compare with the rule for finding the circumference of a circle?

NOTE. — The pupil cannot hope to find the quotient exactly 3.1416, but he should approximate this result.

64. Draw a circle and divide it into 6 equal parts by radii. What is each part approximately? What is the area of a triangle? What is the altitude of each triangle? What is the rule for finding the area of a circle?

65. How far does the outer end of an 8-inch minute hand of a clock pass in one hour? How much surface is passed over by the hand in one hour?

Volumes



The surface on which the prism rests and its opposite surface are called *bases;* the other surfaces, *faces.*

All the surfaces except the bases, as in Ex. 70, make up the *convex surface.* All the surfaces, as in Ex. 71, the *entire surface.*

66. Take a piece of pasteboard and draw two parallel lines 3 in. apart. Lay off on each 4 distances of one inch; connect the points of division. Construct squares upon two opposite sides. Prepare flaps for pasting as represented by the dotted lines. Cut entirely through outside lines and partly through inside lines. Fold and paste the flaps on the inside. You will have a *square prism.*

67. Proceed as before, making the distance between the parallel lines one inch. You will have a *cube.*

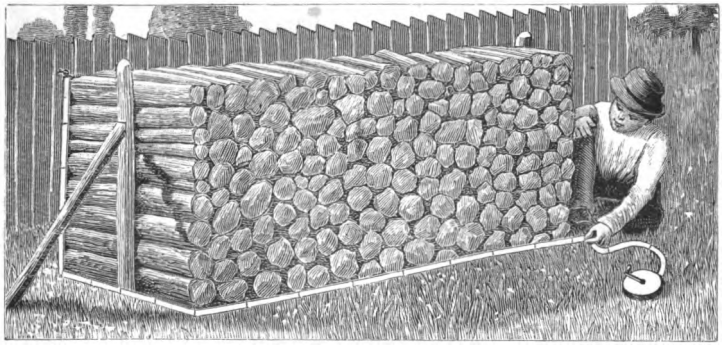
68. Draw a prism whose base is a square.

69. Draw a prism whose base is a triangle.

70. How many square inches of paper would cover all the faces of the prism in Ex. 68 except the squares?

71. How many square inches would cover the whole prism?

Volumes



Solids have *volume*, *i.e.*, they are made up of cubic units. Amount of volume is expressed by the measures in the following table called *Cubic Measure*. See p. 69.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

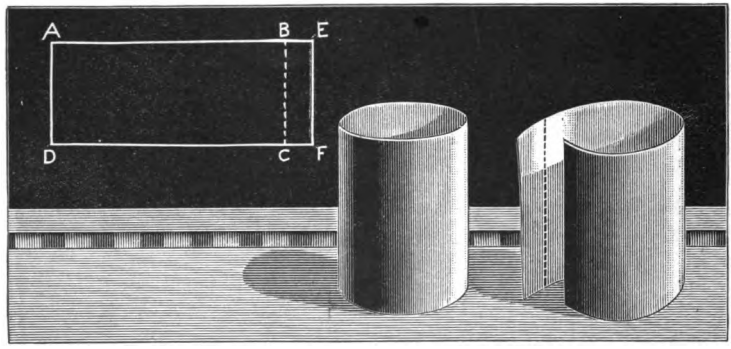
72. Draw a line 3 units long; upon it construct a cube; divide each edge into 3 equal parts to represent feet; draw the cubes in one layer. How many cubes in one layer? How many layers? How many cubic feet make 1 cu. yd.?

73. Draw a line 12 units long; upon it construct a cube; divide each edge of the cube into 12 equal parts to represent inches; draw the cubes in one layer. How many cubes will there be in one layer? How many layers? How many cubic inches make a cubic foot?

How much wood is expressed by the measures in the following table called *Wood Measure?*

16 cubic feet = 1 cord foot (cd. ft.).
8 cord feet, or 128 cubic feet = 1 cord (cd.).

Volumes



The volume of a prism or cylinder is equal to the product of the area of its base by its altitude.

74. What is the volume of the prism constructed in Ex. 66?

Ans. 3 cu. in.; the area of the base is 1 sq. in.; its altitude is 3 in.; its volume is 1 sq. in. × 3 in., or 3 cu. in. Again, there is 1 cube in each layer and there are 3 layers.

NOTE. — The altitude of a prism or cylinder is its height, or perpendicular distance between its bases.

75. How many cubic feet in a pile of wood 8 ft. × 4 ft. × 4 ft.?

Ans. 128 cu. ft. The area of the base is 8 ft. × 4 ft., or 32 sq. ft.; the altitude is 4 ft.; its volume is 32 sq. ft. × 4 ft., or 128 cu. ft., or 1 cord. Again, there are 32 cubes in each layer, and 4 layers.

76. Measure the length, breadth, and height of this room. How many cubic feet of air in the room?

77. Measure the chalk box. How many cubic inches in its volume?

78. Draw a rectangle as in the illustration; roll the rectangle until AD rests on BC, and paste. You will have a cylinder. Compute its volume.

Weight

A substance has a tendency to fall to the earth if unsupported. The amount of this tendency is expressed by the measures in the following tables. See p. 53.

Avoirdupois Weight is used for all ordinary articles; as coal, iron, hay, grain, groceries, etc.

16 ounces (oz.)	= 1 pound (lb.).
100 pounds	= 1 hundredweight (cwt.).
20 hundredweight	= 1 ton (T.).

Troy Weight is used for gold, silver, and jewels.

24 grains (gr.)	= 1 pennyweight (pwt.).
20 pennyweights	= 1 ounce (oz.).
12 ounces	= 1 pound (lb.).

Apothecaries' Weight is used for medicines.

20 grains (gr.).	= 1 scruple.
3 scruples	= 1 dram.
8 drams	= 1 ounce.
12 ounces	= 1 pound (lb.).

NOTE. — The grain is the same for all measures: 5760 gr. = 1 troy, or apothecaries' pound; 7000 gr. = 1 avoirdupois pound.

79. In what stores do they use avoirdupois weight? troy weight? apothecaries' weight?

80. How many ounces in a pound of tea? in a pound of sulphur? in a pound of silver?

81. How many grains in the apothecaries' ounce? the avoirdupois ounce? the troy ounce?

82. Which is the heaviest, a pound apothecaries, a pound troy, or a pound avoirdupois? an ounce apothecaries, an ounce troy, or an ounce avoirdupois?

Capacity

Liquids and small solids, like grains of wheat, must be contained in some receptacle in order to be measured. The amount such a receptacle will hold is expressed by the measures in the following tables. See p. 80.

Liquid Measure is used for liquids; as water, oil, molasses, etc.

4 gills (gi.)	**= 1 pint (pt.).**
2 pints	**= 1 quart (qt.).**
4 quarts	**= 1 gallon (gal.).**

Dry Measure is used for small solids; as grains of wheat, potatoes, etc.

2 pints (pt.)	**= 1 quart (qt.).**
8 quarts	**= 1 peck (pk.).**
4 pecks	**= 1 bushel (bu.).**

NOTE. — A gallon contains 231 cu. in.; a bushel, 2150.4 cu. in.

83. Turn to p. 80. Prepare a liquid quart measure.

84. In a similar manner, prepare the dry quart measure. AB must be $12\frac{1}{8}$ in., AD, $5\frac{3}{4}$ in.

85. Which is the larger, a liquid quart or a dry quart?

86. How many cubic inches in a liquid quart? How many cubic inches in a dry quart?

87. Measure in inches the diameter of the base of the liquid quart measure.

88. By the rule on p. 190 for the volume of a cylinder, compute the volume of the liquid quart measure.

89. How does your answer compare with the true value of a liquid quart, $57\frac{3}{4}$ cu. in.?

90. Capacity is sometimes determined by weighing. If it is found that a pint of water weighs a pound, how many pints in 100 pounds of water? How many gallons?

Time and Value

Amount of *Time* is expressed by the following measures. See p. 54.

60 seconds (sec.)	**= 1 minute (min.).**
60 minutes	**= 1 hour (hr.).**
24 hours	**= 1 day (da.).**
7 days	**= 1 week (wk.).**
30 days	**= 1 business month (mo.).**
365 days	**= 1 common year (yr.).**
366 days	**= 1 leap year (yr.).**
100 years	**= 1 century.**

NOTE. — The year is divided into months of different lengths. Thus: first month, January, 31 days; second, February, 28 or 29 da.; third, March, 31 da.; fourth, April, 30 da.; fifth, May, 31 da.; sixth, June, 30 da.; seventh, July, 31 da.; eighth, August, 31 da.; ninth, September, 30 da.; tenth, October, 31 da.; eleventh, November, 30 da.; twelfth, December, 31 da.

A day is the time of the revolution of the earth upon its axis; a month, the time of the revolution of the moon around the earth; a year, the time of the revolution of the earth around the sun.

Amount of *Money* is expressed by the following measures.

10 mills (m.)	**= 1 cent (¢).**
10 cents	**= 1 dime (d.).**
10 dimes	**= 1 dollar ($).**
10 dollars	**= 1 eagle (E.).**

91. Name the gold coins.

Ans. $20 piece (double eagle), $10 piece (eagle), $5 piece, $1 piece.

92. Name the silver coins.

Ans. Dollar, half dollar, quarter, dime.

93. Of what are nickels made? pennies?

Reduction

94. Reduce 5 bu. 2 pk. 1 pt. to the lowest denomination.

bu.	pk.	qt.	pt.
5	2	0	1
4			
22 pk.			
8			
176 qt.			
2			
353 pt.			

1 bu. is 4 pk.; 5 bu., 5 times 4 pk., or, with 2 pk., 22 pk.

1 pk. is 8 qt.; 22 pk., 22 times 8 qt., or 176 qt.

1 qt. is 2 pt.; 176 qt., 176 times 2 pt., or, with 1 pt., 353 pt.

Ans. 353 pt.

95. Reduce 353 pt. (dry measure) to higher denominations.

2)353 pt.
8)176 qt. 1 pt.
4)22 pk.
5 bu. 2 pk.

2 pt. are 1 qt.; 353 pt., as many qt. as 2 is contained times in 353, or 176 qt., and 1 pt. remaining.

8 qt. are 1 pk.; 176 qt., as many pk. as 8 is contained times in 176, or 22 pk.

4 pk. are 1 bu.; 22 pk., as many bu. as 4 is contained times in 22, or 5 bu., and 2 pk. remaining.

Ans. 5 bu. 2 pk. 1 pt.

Reduce:

96. 13 gal. to gills.
97. 1086 pt. to gallons.
98. 264 pt. to bushels.
99. 1 bu. 1 pt. to pints.
100. 12 gal. 3 qt. to pints.
101. 2 oz. 2 gr. to grains.
102. 1 rd. 2 yd. to inches.
103. 38400 rd. to miles.
104. 7296 gr. to lb. (Troy).
105. 3650 in. to yards.
106. 2112 cu. ft. to cords.
107. 3896 oz. to lb. (avoir.).
108. 75 cu. ft. to cu. yards.
109. 63360 in. to miles.

NUMBERS BY LETTERS

The Signs "+" and "−"

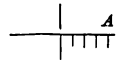
The sign "+" has any meaning that may be agreed upon; the sign "−" denotes the opposite of "+" in the same position.

ILLUSTRATIONS. — In $+6°$, "+" means *above zero* by agreement; in $-6°$, "−" means the opposite, or *below zero*. In $8+6$, "+" means *add* by agreement; in $8-6$, "−" means the opposite, or *subtract*.

1. If "+" means *to the north*, what does $+4$ miles mean? -3 miles? $+5$ miles? -6 miles?

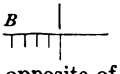
2. If "+" means *to the east*, what does $+6$ rods mean? -3 rods? $+5$ rods? -4 rods?

3. Draw a cross. If "+" means *to the right*, represent $+4$.

Ans. 

A is 4 spaces to the right.

4. In the same way, represent -4.

Ans. 

Since "+ 4" means 4 to the right, "− 4" must mean 4 to the opposite of the right, or 4 to the left. B is 4 spaces to the left.

In the same way, represent:

5. $+6$	8. $+5$	11. -5
6. -6	9. -8	12. $+7$
7. -7	10. -9	13. $+4$

Addition and Subtraction

To add when the signs are alike, write the sum and use the common sign; to add when the signs are unlike, write the difference and use the sign of the greater.

14. Prove that $(+3)+(-5)=-2$.

Ans. Let '+' mean *cash* and '−' the opposite, or *debt.* If John has '+ \$3' or \$3 cash, and James has '− \$5' or a \$5 debt, together they have '− \$2' or a debt of \$2.

15. Prove that $(+3)+(+5)=+8$.

16. Prove that $(-3)+(-5)=-8$.

17. Prove that $(-3)+(+5)=+2$.

	-5	$+5$	$+7$	-8	-9	-6	-8	-7
18. Add:	-6;	-8;	-5;	-3;	$+5$;	-3;	$+8$;	$+9$.

19. What equals '− 3' used 2 times as an addend?

Ans. '− 3' used 2 times as an addend equals $(-3)+(-3)$, or -6.

To subtract, change the sign of the subtrahend and proceed as in addition.

20. Prove that $(-3)-(-5)=+2$.

Ans. The *remainder* plus the *subtrahend* must equal the *minuend;* $(+2)+(-5)=-3$.

21. Prove that $(-3)-(+5)=-8$.

22. Prove that $(+3)-(+5)=-2$.

23. Prove that $(+3)-(-5)=+8$.

	-6	-9	-4	-4	$+4$	$+4$	-2
24. Subtract:	-8;	-3;	-7;	$+7$;	-7;	$+7$;	-8.

25. What equals '− 3' used 2 times as a subtrahend?

Ans. Every time '− 3' is used as a subtrahend, we must change its sign and proceed as in addition. Therefore, '− 3' used once as a subtrahend equals + 3; twice, $(+3)+(+3)$, or + 6.

Multiplication and Division

The product of like signs is "+", the product of unlike signs is "−."

26. Prove that $(+3)\times(+2)=+6$.

Ans. $(+3)\times(+2)$ means that $+3$ is taken 2 times as an addend, or $+3+3$, or $+6$.

27. Prove that $(-3)\times(-2)=+6$.

Ans. $(-3)\times(-2)$ means that -3 is taken 2 times as a subtrahend, or $-(-3)\ -(-3)$, or $+3+3$, or $+6$.

28. Prove that $(-3)\times(+2)=-6$.

Ans. $(-3)\times(+2)$ means that -3 is taken 2 times as an addend, or $-3-3$, or -6.

29. Prove that $(+3)\times(-2)=-6$.

Ans. $(+3)\times(-2)$ means that $+3$ is taken 2 times as a subtrahend, or $-(+3)-(+3)$, or $-3-3$, or -6.

	-3	$+4$	-5	$+3$	-3	$+5$	-5
30. Multiply:	-4;	-5;	$+4$;	$+5$;	-5;	-3;	$+3$.

The quotient of like signs is "+"; the quotient of unlike signs is "−."

31. Prove that $(+6)\div(+2)=+3$.

Ans. The *quotient* multiplied by the *divisor* must equal the *dividend;* $(+3)\times(+2)=+6$.

32. Prove that $(+6)\div(-2)=-3$.

Ans. The *quotient* multiplied by the *divisor* must equal the *dividend;* $(-3)\times(-2)=+6$.

33. In the same way, prove that $(-6)\div(+2)=-3$.

34. In the same way, prove that $(-6)\div(-2)=+3$.

35. State the quotient: $(-8)\div(-2)$; $(+8)\div(+2)$; $(-8)\div(+2)$; $(+8)\div(-2)$; $(-10)\div(-2)$; $(-10)\div(+2)$; $(+10)\div(+2)$; $(+10)\div(-2)$.

Simple Equations

To transpose a term from one member of an equation to the other, change its sign.

ILLUSTRATION. $6 + 8 = 14$. Transposing 6 to the other side of the equation, $8 = 14 - 6$.

36. Transpose to the left-hand member the terms which contain x, and to the right-hand member all other terms, in the equation: $6 + 5x - 8 = 3x + 4$. *Ans.* $5x - 3x = -6 + 8 + 4$.

NOTE. — When no sign is written before the first term, '+' is understood.

37. In the same way, transpose in: $8 + 3x = -5x + 16$.

38. In: $x + 12 = 22$, find the value of x.

$$x + 12 = 22$$

Transposing, $x = 22 - 12$

Uniting, $x = 10$

39. Make a problem for the equation in Ex. 38.

Ans. If 12 is added to a certain number, the sum is 22. Find the number.

40. Prove that 10 is the answer to this problem.

41. In: $6 + 5x - 8 = 3x + 4$, find the value of x.

Transposing, $5x - 3x = -6 + 8 + 4$

Uniting, $2x = 6$

Dividing by 2, $x = 3$

Find the value of x:

42. $3x - 25 = 11$.

43. $2x + 6 = 8$.

44. $5x - 9 = 2x$.

45. $7x - 5 = 3x + 15$.

46. $8x + 6 = 3x + 26$.

47. $5x - 2 = 2x + 13$.

Simple Equations

48. Make a problem: for Ex. 42; for Ex. 47.

49. If 5 times a certain number is increased by 12, the result is 72. What is the number? Prove.

Let x = *the no.*
$5x$ = *5 times the no.*
$5x + 12$ = *5 times the no. incr. by 12*
72 = *the result*
$\therefore\ 5x + 12 = 72$
Transposing, $5x = 72 - 12$
Uniting, $5x = 60$
Dividing by 5, $x = 12$

PROOF. — 5 times the no. increased by 12 must equal 72; 5×12, increased by 12, does equal 72.

50. James has 5 times as much money as John. If James' money is diminished by \$10, he will then have 3 times as much as John. How much has each? Prove.

Let x = *John's, \$ 5*
$5x$ = *James', \$ 25*
$5x - 10$ = *James', less \$ 10*
$3x$ = *3 times John's*
$\therefore\ 5x - 10 = 3x$
$x = 5$

EXPLANATION. — Let x = John's; $5x$ must equal James', because James has 5 times as much as John. $5x - 10 = 3x$, because James' less \$10 = 3 times John's.

51. A father is 3 times as old as his son; if the father were 10 years older he would be 4 times as old as his son. How old is each? Prove. SUGGESTION. — Let x = age of son.

52. A had 3 times as much money as B; A spent \$100 and B earned \$200; A then had the same amount as B. How much had each at first? Prove.

53. During the first year, a merchant doubled his capital; during the second year, he gained \$2000; he then had \$6000. How much had he at the beginning?

SUGGESTION. — Let x = capital at first.

Simple Equations

To clear of fractions, multiply both members of the equation by the same number.

54. Find the value of x: $\frac{3x}{2} - 10 = \frac{2x}{3}$

$\frac{3x}{2} - 10 = \frac{2x}{3}$

$9x - 60 = 4x$

$9x - 4x = 60$

$5x = 60$

$x = 12$

Multiply both members of the equation by the least common denominator, 6.

$\frac{3x}{2} \times 6 = 9x$; $-10 \times 6 = -60$;

$\frac{2x}{3} \times 6 = 4x$.

55. Make a problem for Ex. 54.

Ans. If 10 is subtracted from $\frac{3}{2}$ of a number, the result is $\frac{2}{3}$ of the number. Find the number.

56. Prove that 12 is the answer to Ex. 55.

Find the value of x:

57. $\frac{3x}{4} + \frac{2x}{5} = 23.$

58. $\frac{5x}{3} - \frac{3x}{2} = 2.$

59. $\frac{x}{3} - 1 = \frac{x}{4} + 1.$

60. $\frac{3x}{7} - 7 = \frac{x}{5} + 1.$

61. Make a problem for Ex. 57.

Ans. $\frac{3}{4}$ of John's money increased by $\frac{2}{5}$ of his money is \$23. How much has he?

62. Solve the problem just given.

Let x = John's money

$\frac{3x}{4} = \frac{3}{4}$ *of his money*

$\frac{2x}{5} = \frac{2}{5}$ *of his money*

$\therefore \frac{3x}{4} + \frac{2x}{5} = 23$

etc.

Simple Equations

63. If a man had as many more apples as he has, $\frac{1}{2}$ as many more, and $2\frac{1}{2}$ apples, he would have 100. How many has he?

Let x = *the number of apples*

x = *as many more*

$\frac{x}{2}$ = *half as many more*

$$x + x + \frac{x}{2} + \frac{5}{2} = 100$$

Clearing, $2x + 2x + x + 5 = 200$

Uniting, $5x = 200 - 5$

$5x = 195$

$x = 39$

PROOF:

$$\begin{array}{r} 39 \\ 39 \\ 19\frac{1}{2} \\ 2\frac{1}{2} \\ \hline 100 \end{array}$$

64. Compare the following with the solution just given.

If the number of apples increased by $2\frac{1}{2}$ is 100, before the increase, the number must have been 100 diminished by $2\frac{1}{2}$, or $97\frac{1}{2}$.

If he had as many more, and $\frac{1}{2}$ as many more, he would have $2\frac{1}{2}$ times as many as at first.

Since $2\frac{1}{2}$ times the original number is $97\frac{1}{2}$, the original number must be $97\frac{1}{2} \div 2\frac{1}{2}$, or 39.

65. Which of the two solutions do you prefer?

66. There is a certain fraction whose denominator is 5; if the numerator is increased by 7 and the denominator is diminished by 3, the result is 5. Find the fraction. Prove the answer.

SUGGESTION.— Let $\frac{x}{5}$ = the fraction; $\frac{x+7}{2}$ = the fraction as changed.

67. Henry has \$10 less than Joseph. If Henry had \$1 more, he would have half as much as Joseph. How much has each? Prove.

SUGGESTION. — Let x = Joseph's; $x - 10$ = Henry's.

PERCENTAGE

Terms

In many cases it has become customary to use the term **Per Cent,** written %, in place of hundredths.

ILLUSTRATION. $\frac{6}{100}$, or .06, is often written 6%; read, 6 per cent.

In place of the fractions, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$, their per cent equivalents are often used.

1. Reduce each of the common fractions named above to decimal and per cent equivalents, and memorize the results.

Thus: $\frac{1}{2} = .50 = 50\%$; $\frac{1}{3} = .33\frac{1}{3} = 33\frac{1}{3}\%$, etc. See p. 176.

State rapidly the per cent equivalents:

2. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
3. $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$.
4. $\frac{2}{3}$, $\frac{2}{5}$, $\frac{3}{4}$.
5. $\frac{5}{8}$, $\frac{2}{3}$, $\frac{3}{4}$.
6. $\frac{3}{5}$, $\frac{5}{6}$, $\frac{4}{5}$.
7. $\frac{2}{5}$, $\frac{3}{8}$, $\frac{7}{8}$.
8. $\frac{1}{6}$, $\frac{7}{8}$, $\frac{2}{3}$.
9. $\frac{4}{5}$, $\frac{5}{8}$, $\frac{5}{6}$.
10. $\frac{3}{5}$, $\frac{2}{3}$, $\frac{1}{6}$.

State rapidly the common fraction equivalents:

11. $87\frac{1}{2}\%$, $66\frac{2}{3}\%$, $33\frac{1}{3}\%$.
12. $16\frac{2}{3}\%$, 60%, 25%.
13. $62\frac{1}{2}\%$, 20%, $12\frac{1}{2}\%$.
14. $37\frac{1}{2}\%$, 40%, $33\frac{1}{3}\%$.
15. 75%, 80%, 50%.
16. $37\frac{1}{2}\%$, $83\frac{1}{3}\%$, $87\frac{1}{2}\%$.

Applications

17. What is 5% of 40?

$$\begin{array}{r} 40 \\ .05 \\ \hline 2.00 \end{array}$$

5% of 40 is .05 of 40, or 2.

18. What is $12\frac{1}{2}\%$ of 80?

$$\begin{array}{r} 80 \\ .12\frac{1}{2} \\ \hline 10 \end{array}$$

$12\frac{1}{2}\%$ of 80 is $\frac{1}{8}$ of 80, or 10.

NOTE. — A per cent expression must be reduced to a decimal or to a common fraction, before it can be used as a multiplier.

What is:

19. 12% of 40? **21.** 80% of 24? **23.** $12\frac{1}{2}\%$ of 16?

20. 66% of 80? **22.** $83\frac{1}{3}\%$ of 12? **24.** 50% of 18?

25. A farmer having 400 sheep, sold 6% of them. How many did he sell?

$$\begin{array}{r} 400 = no.\ sheep \\ .06 \\ \hline 24.00 = no.\ sold \end{array}$$

He sold .06 of 400 sheep, or 24 sheep.

26. How many sheep did the farmer keep?

27. Of 480 men, 20% enlisted for the Spanish war. How many men enlisted?

28. A has \$3000; B has 87% as much. How much has B?

29. A dealer bought 900 cattle and sold 75% of them. How many did he keep?

30. A man bought a horse for \$60 and sold him at a gain of 10% on the cost. What was the gain? The selling price?

31. A merchant sells \$30,000 worth of goods and loses 3% of this amount by failures of customers. How much does he lose?

Simple Interest

Interest is money paid for the use of money.

32. What is the interest of $1 for 1 yr. at 6%?

Ans. The interest of $1 for 1 yr. at 6% is .06 of $1, or 6¢.

33. What is the interest of $1 for 1 mo. at 6%?

Ans. Since the interest of $1 for 12 mo. is 6¢, for 1 mo. it is $\frac{1}{12}$ of 6¢, or 5 m.

34. What is the interest of $1 for 1 da. at 6%?

Ans. Since the interest of $1 for 30 da. is 5 m., for 1 da. it is $\frac{1}{30}$ of 5 m., or $\frac{1}{6}$ of a mill.

TO BE MEMORIZED. — *The interest of* $1 *for* 1 *year at* 6% *is* 6¢; *for* 1 *month,* $\frac{1}{2}$ *of a cent; for* 1 *day,* $\frac{1}{6}$ *of a mill.*

35. What is the interest of $1 for 2 yr. 5 mo. 17 da. at 6%?

$.12$
$.025$
$.002\frac{5}{6}$
$.147\frac{5}{6}$

The interest of $1 for 2 yr. at 6% is $.12; for 5 mo., $.025; for 17 da., $.002$\frac{5}{6}$; for the whole time, $.147$\frac{5}{6}$.

Find the interest of $1 *at* 6% *for:*

36. 3 yr. 2 mo. 18 da.

37. 2 yr. 7 mo. 19 da.

38. 6 yr. 9 mo. 27 da.

39. 7 yr. 3 mo. 12 da.

40. 6 yr. 2 mo. 15 da.

41. 5 yr. 5 mo. 5 da.

42. What is the interest of $125 for 1 yr. 3 mo. 7 da. at 6%? The amount?

$125
$.076\frac{1}{6}$
$9.520\frac{5}{6}$, *int.*
$134.52, *amt.*

$.06$
$.015$
$.001\frac{1}{6}$
$.076\frac{1}{6}$

The interest of $1 for 1 yr. 3 mo. 7 da. at 6% is $.076$\frac{1}{6}$; the interest of $125 is 125 times $.076$\frac{1}{6}$, or $9.52.

The amount is the principal plus the interest, or $134.52.

Simple Interest

At 6%, find the interest of $300.50:

43. For 2 yr. 6 mo. 12 da.

44. For 3 yr. 5 mo. 18 da.

45. For 5 yr. 3 mo. 19 da.

At 6%, find the amount of $275.65:

46. For 1 yr. 7 mo. 7 da.

47. For 2 yr. 3 mo. 29 da.

48. For 3 yr. 1 mo. 28 da.

49. After you have found the interest at 6%, how can you modify the result for 7%?

Ans. Add $\frac{1}{6}$ of the interest; 1% is $\frac{1}{6}$ of 6%, and 6% + 1% is 7%.

50. The interest of a certain sum at 6% is $24. What is the interest at 7%?

51. How would you modify the interest at 6% to obtain the interest at 8%?

Ans. Add $\frac{1}{3}$ of the interest; 2% is $\frac{1}{3}$ of 6%, and 6% + 2% = 8%.

52. How would you modify the result at 6% to obtain the interest at 10%? *Ans.* Divide by 6 and multiply by 10.

53. Why not add $\frac{2}{3}$ of the interest?

Ans. This would give the interest at 10%, because $\frac{2}{3}$ of 6% is 4%, and 6% + 4% is 10%, but the process is longer.

54. Find the interest of $125 for 1 yr. 3 mo. 7 da., at 9%.

$9.52, int. at 6%
4.76, int. at 3%
$14.28, int. at 9%

The interest at 6% is $9.52 (see Ex. 42); at 3%, it is $\frac{1}{2}$ of $9.52; etc.

55. Find the interest of $256.75 for 1 yr. 3 mo. 17 da., at 10%.